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Influence of Sampling Interval on the Effect of False Coupling between Oscillators with Different Natural Oscillation Parameters

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Abstract—To analyze the coupling between oscillating systems by time series, the Granger causality assessment—an improved prognosis of the autoregression model—is widely used. It is known that wrong conclusions regarding the presence of bidirectional coupling can be obtained in the case of unidirectional coupled systems when the sampling interval is rather wide. However, it remains unclear under what conditions the effect of false coupling is significant, and thus criteria of significance to account for this effect in practice are absent. In this work, such conditions were studied and qualitatively formulated for an etalon system of coupled oscillators. In particular, it is shown that this effect is negligible in the case of insufficient data if a "fast" oscillator (with a smaller oscillation period and relaxation time) is driving a "slow" oscillator, while the effect is strong otherwise. If both periods are considerably larger than the sampling interval, the effect increases with relaxation time of the driving oscillator and decreases with increasing relaxation time of the driven one.

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A need to quantify direct coupling between oscillating systems by time series (a discrete sequence of observed values) occurs in different areas of physical research. In this case, the question of whether the coupling is unidirectional or bidirectional is important in various problem statements. To answer this question, the Granger causality method is often used. It is based on the use of an prediction improvement (PI) of the common autoregression model relative to the individual autoregression model proposed in econometrics [1] and is more often used in communications [2], geophysics [3], and biophysics [4]. However, nonzero PI can be observed (in both sides) for unidirectional coupled systems at a rather high sampling volume (data discretization in time) [5]. Underestimating this fact can lead to wrong conclusions regarding the coupling, which we call the "false coupling effect" (FCE). To practically determine whether the use of additional rather cumbersome and complex tests [6] in order to avoid FCE is necessary or not and when this effect can be neglected, we study in this paper the properties of oscillating systems for which FCE is deliberately set to be insignificant or significant.

Let the time series $\{x_{1,n}\}$ and $\{x_{2,n}\}$ of processes $x_1(t)$ and $x_2(t)$ be observed where $x_{k,n} = x_k(n\Delta t)$, $k = 1, 2$; n is the integer number; and Δt the sampling time interval. When assessing the linear Granger causality, we first consider the following individual autoregressive representation of the processes studied

$$x_k(t) = A_{k,0} + \sum_{i=1}^{\infty} A_{k,j} x_k(t-i) + \xi_k(t), \quad k = 1, 2, \tag{1}$$

where ξ_k is normal white noise. The ξ_k dispersion is denoted σ_k^2. It equals the rms error corresponding to one-step prediction of model (1). In addition, we consider a combined model

$$x_k = a_{k,0} + \sum_{i=1}^{\infty} a_{k,j} x_k(t-i) + \sum_{i=1}^{\infty} b_{k,j} x_j(t-i) + \eta_k(t), \quad k, j = 1, 2, \quad j \neq k, \tag{2}$$

where η_1, η_2 is the two-dimensional white noise. The η_k dispersion is denoted as $\sigma_{k|j}^2$. The normalized value of PI is $G_{j\to k} = (\sigma_k^2 - \sigma_{k|j}^2)/\sigma_k^2$. It characterizes the action $j \longrightarrow k$ and takes values from 0 to 1. Representations (1) and (2) exist and are unique for stationary Gaussian processes. In practice, finite order models (1) and (2) are used to determine theoretical characteristics $G_{j\to k}$, while the coefficients are calculated by the least squares method [1, 7]. If $G_{j\to k}$ is positive, then the direct approach leads to the conclusion that action $j \longrightarrow k$ is present.

However, situations are possible in which the coupling is unidirectional ($j \longrightarrow k$), but the PI corresponding to the "false" side is positive ($G_{k\to j} > 0$), being rather

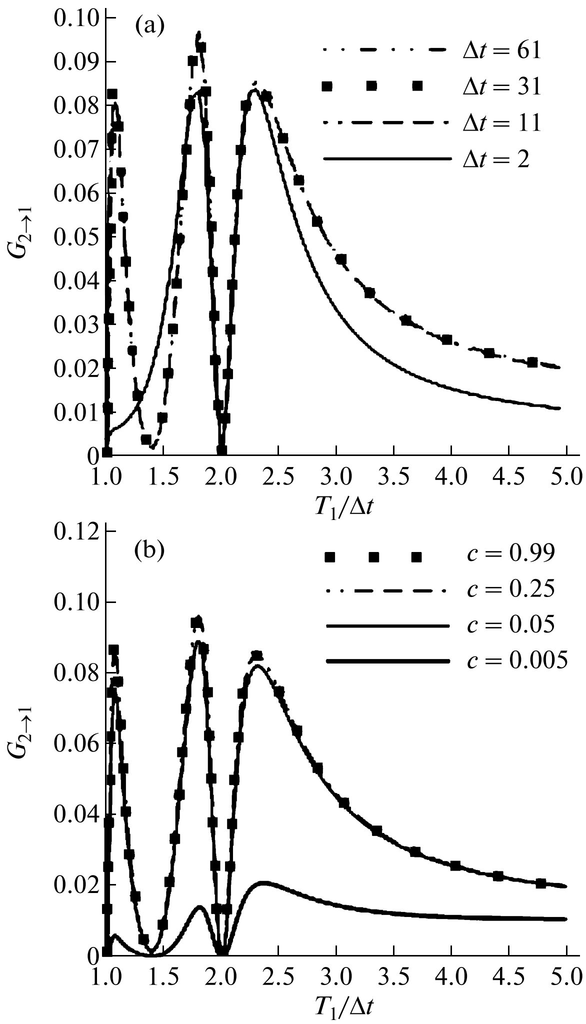

Fig. 1. "False side" PI values for system (3) at $\tau_1 = \tau_2 = 8\Delta t$, $T_1 = T_2$, $\sigma_1^2 = \sigma_2^2 = 1$: (a) dependences $G_{2\to1}(T_1/\Delta t)$ at $c = 0.5$ and different ΔT; (b) dependences $G_{2\to1}(T_1/\Delta t)$ at $\Delta t = 11$ and different c.

high [5, 6]. The higher theoretical magnitude $G_{k\to j}$, the shorter the time series where it can be thought to be definitely different from zero (when the series contains about $4/G_{k\to j}$ members, as follows from the Fisher test [8]). This will result in the "false" conclusion that action $k \longrightarrow j$ is present. In this work, we assume that FCE is strong in the situation in which the magnitude of PI on the "false" side exceeds 0.05, while this effect is weak when the PI is less than 0.005.

Let us examine the conditions of strong and weak FCE using an etalon system of stochastic linear dissipative oscillators described by the difference equations [9]

$$\begin{aligned} x_1(t) &= a_1x_1(t-1) + b_1x_1(t-2) + \zeta_1(t), \\ x_2(t) &= a_2x_2(t-1) + b_2x_2(t-2) + \zeta_2(t) + cx_1(t-1), \end{aligned} \tag{3}$$

where t is the discrete time, c is the coupling coefficient, $a_k = 2\cos(2\pi/T_k)\exp(-1/\tau_k)$; $b_k = -\exp(-2/\tau_k)$; $k = 1, 2$; T_k are natural oscillation periods; τ_k is the relaxation times; and ζ_k is the mutually correlated Gaussian white noises with dispersions σ_k^2. Series x_1 and x_2 with sampling interval $\Delta t > 1$ are observed, i.e., decimated time series.

This simple and rather general model of oscillation processes allows one to carry out exact calculation of the PI using covariance matrices, which are calculated by solving linear difference equations of moments [10], instead of the ordinary estimates using long time series. Below, we obtain values of "false" coupling $G_{2\to1}$ at different values of c, Δt, oscillation periods, and relaxation times that describe a wide number of practical situations. The results obtained can be largely extended for nonlinear oscillators, where the idea of Granger causality can be generalized using nonlinear models and when the FCE is quite similar [11].

For the same ratios of timescales $\tau_1/\tau_2 = T_1/T_2$, i.e., the same quality factor of oscillations, we consider consecutively the characteristic cases of identical oscillators ($T_1 = T_2$), a more rapid driving oscillator ($T_1 < T_2$), and a less rapid driving oscillator ($T_1 > T_2$).Dependence $G_{2\to1}$ versus $T_1/\Delta t$ corresponding to the first case is shown in Fig. 1a for different Δt and relaxation times considerably exceeding Δt. The results only slightly differ one from another at any $\Delta t > 10$, and this is the case for oscillators with different parameters. Therefore, in what follows, all the results are given for $\Delta t = 11$. This corresponds to typical situations in which system (3) is an adequate difference scheme at a small time step, whereas the observations are performed for considerably wider interval Δt. As for c, the values of $G_{2\to1}$ are small only at very small *effective* coupling parameters $c_{\text{eff}} = c\sigma_1/\sigma_2$ (Fig. 1b, thick line); they rapidly increase with increasing c and become saturated at $c_{\text{eff}} = 0.25$, despite the maximum value of the mutual correlation function of $x_1(t)$, $x_2(t)$ not being very high (it takes values from 0.02 to 0.41 at $c_{\text{eff}} = 0.25$, depending on $T_1/\Delta t$). In the following, we take $\sigma_1 = \sigma_2 = 1$ and $c = 0.5$ to obtain the most complete information on various situations relating to the role of FCE.

Figure 1 shows that the value of $G_{2\to1}$ can be high: it reaches nearly 0.1 at $T_1/\Delta t \approx 1.7$ and exceeds 0.05 at $1.05 \le T_1/\Delta t \le 1.15$, $1.6 \le T_1/\Delta t \le 1.9$, and $2.1 \le T_1/\Delta t \le 2.6$. Multiple local maxima with high $G_{2\to1}$ appear for very poor sampling $T_1/\Delta t < 1$. This particular case is not shown in individual figures. Figure 2 shows that, in comparison to the case of identical parameters, the FCE becomes more typical and strong when a "slow" oscillator drives a "rapid" one (Fig. 2a, thick solid line, $T_2/T_1 = \tau_2/\tau_1 = 8$). In the opposite case, the FCE is weak and negligible (Fig. 2b, thick solid line, $T_2/T_1 = \tau_2/\tau_1 = 0.1$). These conclusions refer to

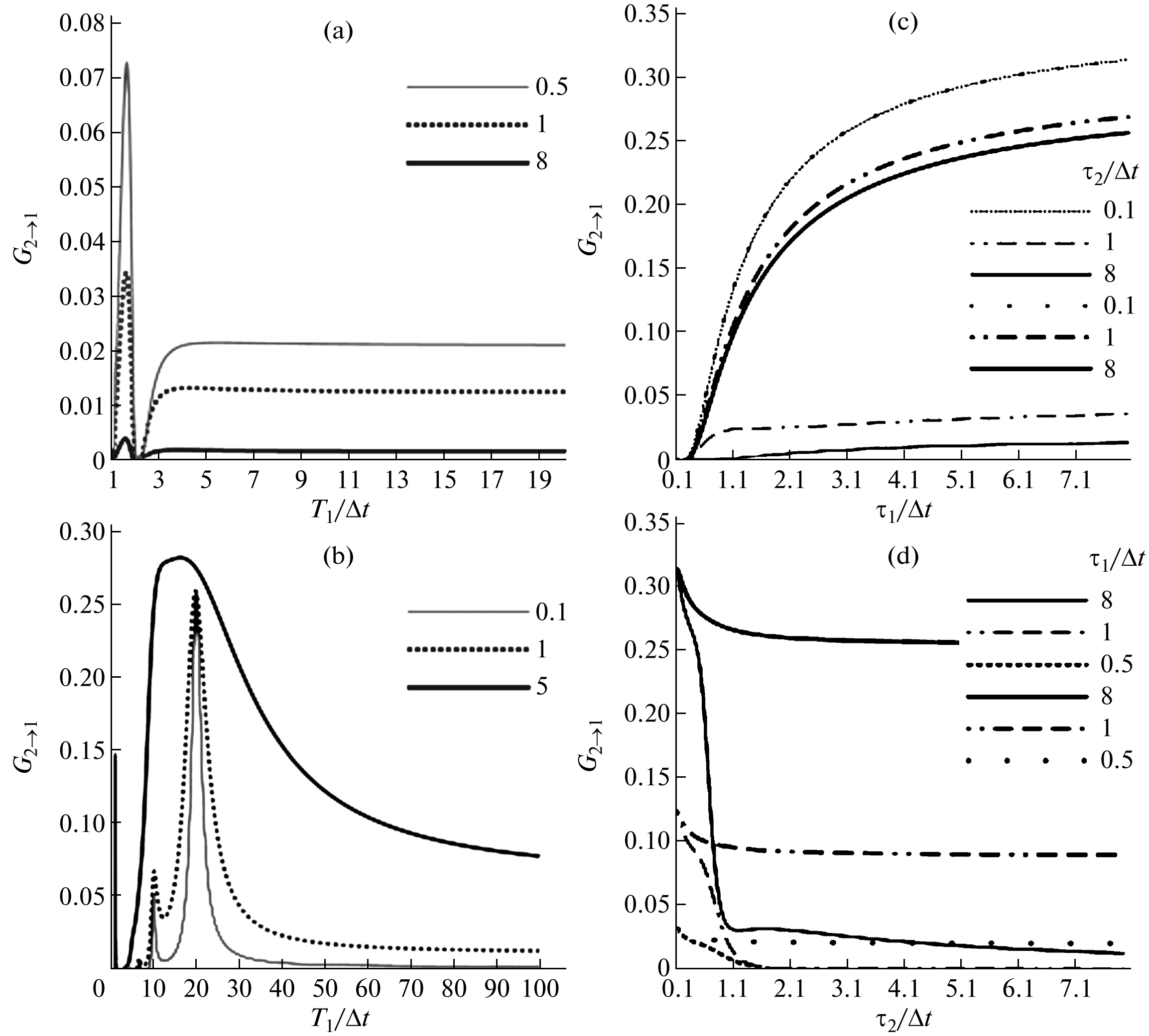

Fig. 2. Same as in Fig. 1 for system (3) with different periods and relaxation times at $c = 0.5$, $\Delta t = 11$, $\sigma_1^2 = \sigma_2^2 = 1$: (a) $G_{2\to1}(T_1/\Delta t)$ at $T_2/T_1 = 8$, $\tau_1 = 8\Delta t$ and $\tau_1/\tau_2 = 8$ (thick solid line), $\tau_1/\tau_2 = 1$ (dotted line), $\tau_1/\tau_2 = 0.5$ (thin solid line); (b) $G_{2\to1}(T_1/\Delta t)$ at $T_2/T_1 = 0.1$, $\tau_1 = 8\Delta t$ and $\tau_1/\tau_2 = 0.1$ (thick solid line), $\tau_1/\tau_2 = 1$ (dotted line), $\tau_1/\tau_2 = 5$ (thin solid line); (c) $G_{2\to1}(\tau_1/\Delta t)$ at $T_1 = 20\,\Delta t$, $T_2 = 2\Delta t$ (thick lines and circles) and $T_1 = 100\Delta t$ and $T_2 = 10\Delta t$ (thin and dotted lines) for different $\tau_1/\Delta t$; and (d) $G_{2\to1}(\tau_2/\Delta t)$ at $T_1 = 20\Delta t$ and $T_2 = 2\Delta t$ (thick solid lines and circles) and $T_1 = 100\Delta t$ and $T_2 = 10\Delta t$ (thin lines and dotted line) for various $\tau_1/\Delta t$.

the most interesting case, in which the sampling interval is slightly less than at least one of the natural periods.

For different quality factors of oscillators (i.e., $\tau_1/\tau_2 \neq T_1/T_2$), it is necessary to discriminate different PI dependences as the functions of relaxation time and periods. Thus, $G_{2\to1}$ increases with decreasing relaxation time (dotted and thin solid lines τ_2 in Fig. 2a), violating both condition $G_{2\to1} < 0.005$ and $G_{2\to1} < 0.05$. Likewise, the effect of false coupling as a whole significantly decreases with increasing τ_2 (Fig. 2b) and remains valid only at a special ratio between the period and sampling interval $T_2 \approx 2\Delta t$ (at $T_1 \approx 20\Delta t$). Thus, the effect of the relaxation time of the driven oscillator on FCE is significant. From Figs. 2c, 2d (thin lines), we can see that this effect is most significant at large τ_1 (in particular, see Fig. 2c on the right side of the abscissa axis and thin solid line in Fig. 2d). The largest difference in the values of $G_{2\to1}$ occurs when passing from $\tau_2/\Delta t \ll 1$ to $\tau_2/\Delta t > 5$. This is most noticeable at large τ_1 (the thin solid line in Fig. 2d). Dependence $G_{2\to1}$ versus τ_1 has the opposite character ($G_{2\to1}$ increases with increasing τ_1, Fig. 2c). This is most noticeable at $\tau_2/\Delta t \ll 1$ (Fig. 2c, the graphs on the top) when passing from $\tau_1/\Delta t \ll 1$ to $\tau_1/\Delta t \approx 1$, becoming weaker at $\tau_1/\Delta t \approx 1$ and $\tau_1/\Delta t \approx 10$ and being saturated at large values of τ_2. These dependences can be distorted for certain small ratios between the periods and sampling intervals—for example, at the aforementioned ratio $T_2 \approx 2\Delta t$ (thick lines and circles in Figs. 2c, 2d). If all parameters τ_1, τ_2, T_1, and T_2 considerably exceed Δt, then $G_{2\to1}$ depends mainly on τ_2 and monotonously decreases with increasing this parameter (thin solid line in Fig. 2d).

Thus, conditions of strong FCE for the model system of unidirectional coupled stochastic oscillators caused by an insufficient sampling volume of the observed time series were studied. It is shown that, for oscillators with the same quality factor. this effect is strong even for poor sampling statistics, when the slow oscillator drives the fast one. In the reverse situation, the effect is negligible. In the first case, the conclusions based on the Granger causality assessment, i.e., the conclusions on the driving effect of fast processes comparing to slow processes, should be additionally checked with the help of special tests [6]. The results in the second case, in which fast processes drive slow processes, can be accepted as a reliable manifestation of the physical action. A strong dependence of this effect on the relaxation times of oscillators is found. Since, in practice, there is often a priori information on the characteristic times of the oscillatory processes studied, the obtained conclusions can be used as a practical criterion if we need to consider the effect of an insufficiently wide sampling in evaluating unidirectional coupling from observational data.

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