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Intermittency of intermittencies

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A phenomenon of intermittency of intermittencies is discovered in the temporal behavior of two coupled complex systems. We observe for the first time the coexistence of two types of intermittent behavior taking place simultaneously near the boundary of the synchronization regime of coupled chaotic oscillators. This phenomenon is found both in the numerical and physiological experiments. The laws for both the distribution and mean length of laminar phases versus the control parameter values are analytically deduced. A very good agreement between the theoretical results and simulation is shown. © *2013 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4819899]

Intermittent behavior occurs widely in nature. At present, several types of intermittency are known and well studied. However, the investigation of intermittency was limited to the analysis of the cases where only one certain type of intermittency takes place. In this paper, we report for the first time on a new temporal behavior of two coupled complex systems, when two types of intermittent behavior coexist and alternate with each other. In other words, we report on intermittency of intermittencies. We demonstrate the presence of this phenomenon both in the numerical and physiological experiments.

Intermittent behavior is a typical phenomenon in nature, which has been found in a great number of nonlinear systems. At the present time, the origin and main statistical properties of intermittent behavior have been studied. Different types of intermittency have been classified as types I, II, and III intermittencies,[1,2] on–off intermittency,[3–5] eyelet intermittency,[6] and ring intermittency.[7] In the case of intermittencies of types I–III, the time intervals of periodic oscillations alternate with the stages of chaotic motion. Similarly, for the eyelet and ring intermittencies, the epochs of synchronized oscillations are interchanged with the epochs of asynchronous motion. However, for any kind of intermittency reported hitherto, one *dynamical regime* always alternates with another *dynamical regime*. These regimes can be of different type, for example, a stationary state, bursts, periodic, or even chaotic oscillations.

From the theoretical point of view, one can imagine that two intermittencies of different (or the same) types can alternate with each other and expect that this phenomenon may be observed in a broad range of different systems, and, as a consequence, be generic. Nevertheless, such kind of system dynamics has not been found out and described hitherto anywhere. In this paper, we report for the first time on a new temporal behavior of two coupled complex systems, when *two types of intermittent behavior* alternate with each other. In other words, in this case, we deal with *the intermittency of intermittencies*, a novel type of intermittent behavior that differs greatly from the ones known so far. Although this phenomenon has not been reported until now, we show below that such kind of behavior is typical for a wide class of systems and can be observed under different circumstances.

In the case of *intermittency of intermittencies*, the distribution of the laminar phase lengths has the following form:

$$p(\tau) = \frac{1}{T_1+T_2}\left[\int_\tau^\infty \frac{ds}{s}\int_\tau^\infty [p_1(l)p_2(s)T_2 + p_1(s)p_2(l)T_1]\,dl + \int_\tau^\infty \left(1-\frac{\tau}{s}\right)[p_1(\tau)p_2(s)T_2 + p_1(s)p_2(\tau)T_1]\,ds\right], \quad (1)$$

where $p_{1,2}(\xi)$ are the distributions of the laminar phase lengths of the alternating intermittencies considered separately and $T_{1,2} = \int_0^\infty s p_{1,2}(s)\,ds$ are the mean lengths of laminar phases for these intermittencies.

To deduce Eq. (1), we consider two different types of intermittency, say, the type-1 and type-2 intermittencies characterized by the distributions $p_1(s)$ and $p_2(s)$, respectively. Let us assume that these types of intermittent behavior coexist in a certain range of the control parameter values. It is supposed that the occurrence of turbulent phases of type-1 intermittency does not depend on the turbulent phases of type-2 intermittency and visa versa. The occurrence of turbulent phases of each type of intermittency is assumed to be determined only by the distribution of the laminar phase lengths of the corresponding intermittency.

Let $p(\tau)$ be the distribution of the laminar phase lengths in the case of intermittency of intermittencies, where τ is the length of laminar phase. At the arbitrary choice of the phase slip without the loss of generality, we can suppose that it corresponds to the type-1 intermittency. Then, the probability to observe the laminar phase with a length falling in the range $[\tau; \tau + d\tau)$ will be defined by a sum of probabilities of two events. The first event is associated with the phase slip of type-2 intermittency within the time $t \in [\tau; \tau + d\tau)$ and described by the probability

$$P_{12}(\tau) = d\tau \int_{\tau}^{\infty} \frac{p_2(s)}{s} ds \int_{\tau}^{\infty} p_1(l)\, dl. \tag{2}$$

The probability of the second event is associated with the phase slip of type-1 intermittency within the same time interval t and described by the probability

$$P_{11}(\tau) = p_1(\tau)\, d\tau \int_{\tau}^{\infty} \left(1 - \frac{\tau}{s}\right) p_2(s)\, ds. \tag{3}$$

Similarly, if the chosen phase slip corresponds to the type-2 intermittency, the probability to observe the phase slips associated with the type-1 and type-2 intermittencies will have the form

$$P_{21}(\tau) = d\tau \int_{\tau}^{\infty} \frac{p_1(l)}{l} dl \int_{\tau}^{\infty} p_2(s)\, ds \tag{4}$$

and

$$P_{22}(\tau) = p_2(\tau)\, d\tau \int_{\tau}^{\infty} \left(1 - \frac{\tau}{l}\right) p_1(l)\, dl, \tag{5}$$

respectively.

The probability that the arbitrary chosen phase slip corresponds to the type-1 intermittency is

$$P_1 = \frac{N_1}{N_1 + N_2} = \frac{T_2}{T_1 + T_2}, \tag{6}$$

where $N_i = L/T_i$ $(i = 1, 2)$ is the number of turbulent slips of the ith type observed in the time series of length L. Similarly, the probability that the arbitrary chosen phase slip corresponds to the type-2 intermittency is

$$P_2 = \frac{N_2}{N_1 + N_2} = \frac{T_1}{T_1 + T_2}. \tag{7}$$

Taking into account all the above mentioned arguments, the probability to observe the laminar phase of the length falling in the range $[\tau; \tau + d\tau)$ takes the following form:

$$\begin{aligned} P(\tau) &= p(\tau)\, d\tau \\ &= P_1[P_{12}(\tau) + P_{11}(\tau)] + P_2[P_{21}(\tau) + P_{22}(\tau)]. \end{aligned} \tag{8}$$

Substituting Eqs. (2)–(7) in Eq. (8), one obtains Eq. (1).

We expect that the intermittency of intermittencies is inherent in different types of intermittent behavior; but in this paper, we report on observation of this phenomenon for the eyelet, type-I with noise, and ring intermittencies.

The intermittency of intermittencies takes place in the certain ranges of time scales and coupling strength values of interacting oscillators. To illustrate this phenomenon and show its universality, we consider several systems, namely, the human cardiovascular system, the Van der Pol oscillator

$$\ddot{x} - (\lambda - x^2)\dot{x} + x = A \sin(\omega_e t) + D\xi(t) \tag{9}$$

driven by the external harmonic signal with the amplitude A and frequency ω_e with the additive stochastic term $D\xi(t)$, and two coupled chaotic Rössler oscillators

$$\begin{aligned} &\dot{x}_1 = -\omega_1 y_1 - z_1, \quad \dot{x}_2 = -\omega_2 y_2 - z_2 + \varepsilon(x_1 - x_2), \\ &\dot{y}_1 = \omega_1 x_1 + a y_1, \quad \dot{y}_2 = \omega_2 x_2 + a y_2, \\ &\dot{z}_1 = p + z_1(x_1 - c), \quad \dot{z}_2 = p + z_2(x_2 - c), \end{aligned} \tag{10}$$

where (x_1, y_1, z_1) and (x_2, y_2, z_2) are the Cartesian coordinates of the drive and the response oscillators, respectively, and ε is a parameter characterizing the coupling strength. Since the behavior of the systems (9) and (10) is quite similar from the point of view of intermittency of intermittencies, we illustrate this phenomenon with the more complex system (10), synchronization in which has been discussed in many papers, e.g., Refs. 8–10.

The intermittency of intermittencies takes place near the origin of the time scale synchronization regime.[11] Time scale synchronization means the presence of the synchronous dynamics in the certain range $[s_l; s_h]$ of the time scales s, introduced with the help of the continuous wavelet transform[12] $W(s, t_0) = s^{-1/2} \int_{-\infty}^{+\infty} x(t) \psi^*\left(\frac{t - t_0}{s}\right) dt$, with Morlet mother wavelet function $\psi(\eta) = \pi^{-1/4} \exp(j2\pi\eta) \exp(-\eta^2/2)$. Each of the time scales can be characterized by the phase $\varphi(s, t) = \arg W(s, t)$. Near the boundary of the time scale synchronization regime, the dynamics of the phase difference $\Delta\varphi_s(t)$ exhibits time intervals of phase synchronized motion (laminar phases) persistently and intermittently interrupted by sudden phase slips (turbulent phases) during which the value of $|\Delta\varphi_s(t)|$ jumps up by 2π. Depending on the system under study and selected values of the control parameters, the observed intermittent behavior may be the eyelet intermittency, type-I with noise intermittency, or ring intermittency,[13] and, as we show below, two different types of intermittent behavior may alternate with each other in some regions of the parameter values.

The typical dependence of the phase difference $\Delta\varphi_s$ on time for two coupled Rössler systems (10) is shown in Fig. 1 for the case of intermittency of intermittencies regime at $\varepsilon = 0.032$ and $s = 4.99$. The intermittency of intermittencies was studied by the analysis of time intervals τ_i (laminar phase lengths) between the consequent turbulent events (slips of eyelet intermittency or ring intermittency). From Fig. 1, one can see that it is hard to distinguish visually the difference between the phase slips corresponding to the different types of intermittency. The simplest way to reveal the intermittency of intermittencies is to use the rotating plane approach.[7,13] For example, for two coupled Rössler systems (10) with $a = 0.15$, $p = 0.2$, $c = 10.0$, $\omega_2 = 0.95$, and $\omega_1 = 0.93$, the boundary of the time scale synchronization regime occurs at $\varepsilon_c \approx 0.045$, with the boundaries of the range of synchronous time scales being $s_l = 4.99$ and $s_h = 8.25$. If we consider $\tilde{x}_{1,2} = \mathrm{Re}\, W_{1,2}(s, t)$ and $\tilde{y}_{1,2} = \mathrm{Im}\, W_{1,2}(s, t)$ as the variables determining the system state, then in the rotating plane (x', y') with[13] $x' = \tilde{x}_1 \cos\varphi_2(s, t) + \tilde{y}_1 \sin\varphi_2(s, t)$ and $y' = -\tilde{x}_1 \sin\varphi_2(s, t) + \tilde{y}_1 \cos\varphi_2(s, t)$, the ring intermittency can be separated easily from the eyelet intermittency. According to the rotating plane concept, the tangle of phase trajectories in the (x', y') plane, which looks

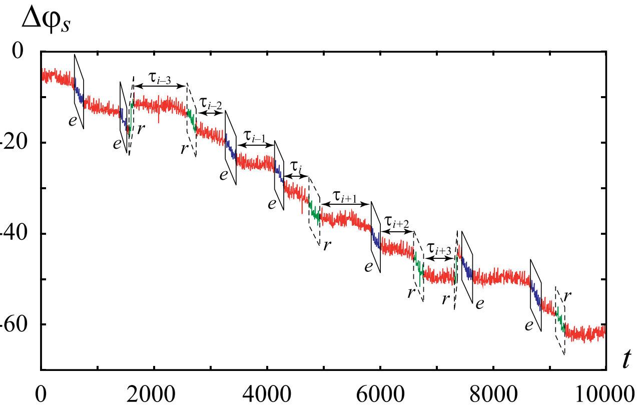

FIG. 1. Typical phase difference $\Delta\varphi_s(t)$ for two coupled Rössler systems in the regime of intermittency of intermittencies at $\varepsilon = 0.032$ and $s = 4.99$. The turbulent phases are shown by rhombuses. The turbulent (asynchronous) regions are denoted by "e" for the slips of eyelet intermittency and by "r" for the slips of ring intermittency.

like a fixed point smeared with noise (Fig. 2(a)), corresponds to the synchronous behavior. When the trajectory starts rotating around the origin (Fig. 2(b)), it is the evidence of the eyelet intermittency presence;[6] whereas in the regime of ring intermittency, the phase trajectory envelops the origin (see Ref. 7 for detail) as it is shown in Fig. 2(c).

From Figs. 2(a)–2(c), one can easily see that at some values of time scale s and coupling strength ε either the eyelet (Fig. 2(b)) or ring (Fig. 2(c)) intermittency takes place. However, at the certain time scales, the eyelet and ring intermittencies exist simultaneously (Fig. 2(d)). It means that eyelet intermittency interrupts ring intermittency, and vice versa, i.e., *the intermittency of intermittencies* takes place. In this regime, the phase trajectory in the (x', y') plane rotates around the origin that is the manifestation of eyelet intermittency and, from time to time, envelops it giving the evidence of the ring intermittency presence. The same results can be

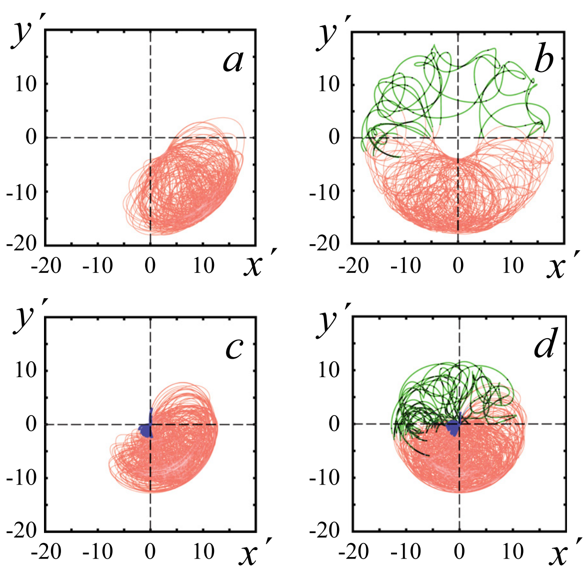

FIG. 2. The phase trajectory of the Rössler system in the (x', y') plane rotating around the origin: (a) synchronous regime at $\varepsilon = 0.045$ and $s = 5.50$, (b) eyelet intermittency at $\varepsilon = 0.032$ and $s = 5.50$, (c) ring intermittency at $\varepsilon = 0.045$ and $s = 4.99$, and (d) intermittency of eyelet and ring intermittencies at $\varepsilon = 0.032$ and $s = 4.99$.

obtained for a driven Van der Pol oscillator in the presence of noise, where the type-I intermittency with noise and ring intermittency are observed. Note, that eyelet intermittency and type-I intermittency with noise are closely related to each other.[14]

For the eyelet, type-I with noise and ring intermittencies the distribution of the lengths of laminar phases is governed by the exponential law $p_{1,2}(\tau) = (1/T_{1,2})\exp(-\tau/T_{1,2})$, where the subscript 1 is used for the ring intermittency and the subscript 2 is used for the eyelet or type-I with noise intermittency. As the result, for the intermittency of intermittencies, Eq. (1) takes the form

$$p(\tau) = \frac{\exp(-\tau/T_1)}{(T_1+T_2)}\left(1-\frac{\tau}{T_1}\right)\Gamma\left(0,\frac{\tau}{T_2}\right) + \frac{T_1^2+T_2^2}{T_1T_2(T_1+T_2)}\exp\left(-\frac{\tau}{T_1}-\frac{\tau}{T_2}\right) + \frac{\exp(-\tau/T_2)}{(T_1+T_2)}\left(1-\frac{\tau}{T_2}\right)\Gamma\left(0,\frac{\tau}{T_1}\right), \tag{11}$$

where $\Gamma(a, z)$ is the incomplete gamma function. The mean length of laminar phases for this type of intermittent behavior can be obtained as

$$\langle\tau\rangle = -\frac{T_1^2\log\left(\frac{T_1+T_2}{T_1}\right) - 2T_1T_2 + T_2^2\log\left(\frac{T_1+T_2}{T_2}\right)}{T_1+T_2}. \tag{12}$$

The mean lengths of laminar phases of the considered types of intermittencies are known to depend on the control parameters. Therefore, substituting the corresponding dependencies for T_1 and T_2 in (12), one can reveal the dependence of the mean length $\langle\tau\rangle$ of laminar phases on the control parameters for the examined type of the behavior. It can be also shown that the probability distribution (11) obeys the normalization condition $\int_0^{+\infty} p(\tau)\, d\tau = 1$.

The probability distributions of the laminar phase lengths obtained numerically for the driven Van der Pol oscillator with noise (9) at the control parameter values $\lambda = 0.1$, $\omega_e = 0.98$, and $D = 1.0$ (Eq. (9) was integrated using the one-step Euler method with the time step $h = 5\times10^{-4}$) and for two coupled Rössler systems (10) in the regime of intermittency of intermittencies are shown in Fig. 3. Since one can distinguish between the cases of eyelet (or type-I with noise) and ring intermittencies, which result in the phase slips, the values of T_1 and T_2 can be estimated from the analyzed time series and used to compare the obtained distribution $p(\tau)$ with the theoretical law (11). The theoretical probability distributions $p(\tau)$ corresponding to these values of T_1 and T_2 are also shown in Fig. 3 by solid lines. One can see that the obtained distributions of the laminar phase lengths agree well with the theoretical prediction (11). Note that a very good agreement between the theoretical curves and numerically obtained data is observed for different values of the control parameters. The dependencies of the mean length $\langle\tau\rangle$ of laminar phases on the control parameter values closely match the theoretically deduced relation (12) for the examined intermittent regimes (Fig. 4).

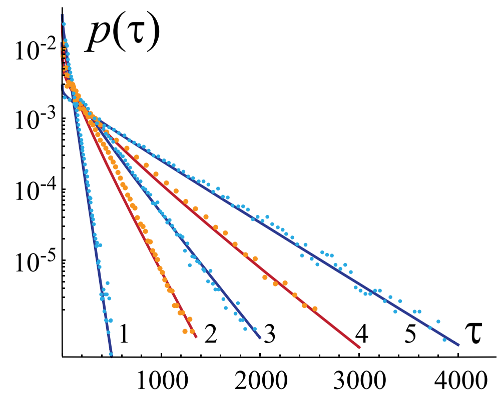

FIG. 3. The laminar phase length distributions for the regime of intermittency of intermittencies in two coupled Rössler systems (curves 1, 3, and 5) and driven Van der Pol oscillator with noise (curves 2 and 4). The theoretical curves (11) are shown by solid lines and the numerically obtained data are shown by dots. The ordinate axis is selected in the log-scale. Curve 1 – $\varepsilon = 0.036$, $s = 4.870$, $T_1 = 50.0$, and $T_2 = 1666.7$; 2 – $A = 0.02308$, $s = 3.50$, $T_1 = 56.0$, and $T_2 = 148.1$; 3 – $\varepsilon = 0.036$, $s = 4.965$, $T_1 = 322.6$, and $T_2 = 1666.7$; 4 – $A = 0.02497$, $s = 3.57$, $T_1 = 120.0$, and $T_2 = 6002.9$; 5 – $\varepsilon = 0.038$, $s = 4.980$, $T_1 = 540.5$, and $T_2 = 17921.1$.

Finally, to prove the generality of our findings, we consider briefly the experimental results of the physiological data analysis. Recently, it has been shown[15,16] that physiological systems generating the main heart rhythm and the rhythm associated with slow oscillations in heart rate can be regarded as self-sustained oscillators, and that the respiration can be regarded as an external forcing of these systems. At the certain values of the breathing frequency, the rhythmic processes operating within the cardiovascular system can be synchronized.[16] Therefore, the phenomenon of intermittency of intermittencies is assumed to be observed in the vicinity of the region of synchronization.

We studied eight healthy young male subjects having average levels of physical activity. The study protocol was approved by the institutional ethical board and all subjects gave their written informed consent. The signals of electrocardiogram (ECG) and respiration were simultaneously recorded in the sitting position with the sampling frequency 250 Hz and 16-bit resolution. The duration of experiments was 10 min. The rate of breathing was set by sound pulses. Having taken into account that the process of slow regulation of heart rate is characterized in humans by the fundamental frequency close to 0.1 Hz,[17] the experiments were carried out under paced respiration with the breathing frequency of 0.2 Hz to avoid the spurious synchronization caused by the presence of the respiratory component in the heart rate variability.[18] As a consequence, the intermittent behavior has been studied near the 1:2 synchronization tongue with the phase difference defined as $\Delta\varphi_s(t) = \varphi_b(s,t) - 2\varphi_{RR}(2s,t)$, where

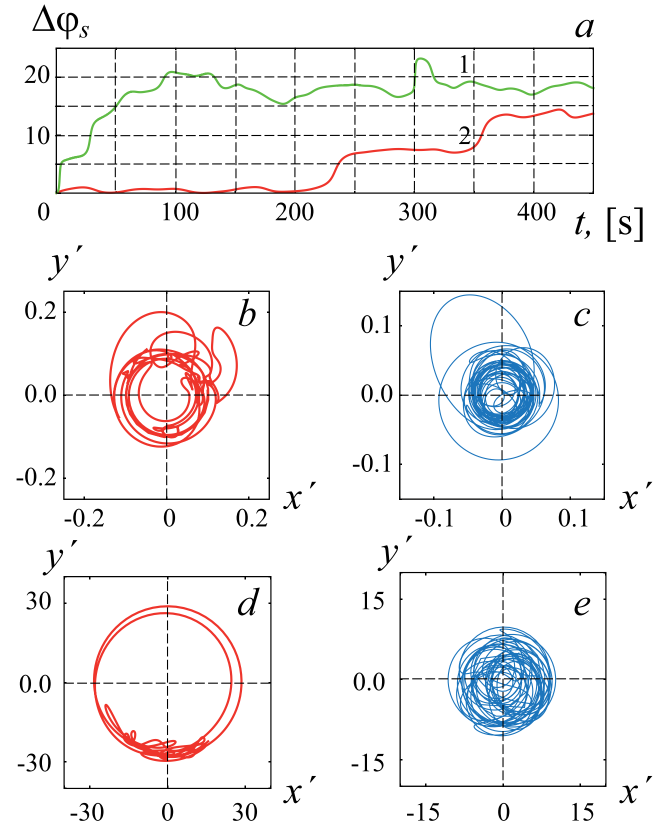

FIG. 5. Intermittency of intermittencies in the human cardiovascular system: (a) the dependence $\Delta\varphi_s(s = 5\ \text{s}, t)$, curve 1, (b) the trajectory in the rotating plane (x', y') revealing the phase slips of eyelet intermittency, $s = 5.0$ s, and (c) the trajectory in the rotating plane (x', y'), where the intermittency of eyelet and ring intermittencies is observed, $s = 4.0$ s. Intermittency of intermittencies in two coupled Rössler systems at $\varepsilon = 0.035$: (a) the dependence $\Delta\varphi_s(s = 6.3, t)$, curve 2, (d) the trajectory in the rotating plane (x', y') revealing the phase slips of eyelet intermittency, $s = 6.3$, and (e) the trajectory in the rotating plane (x', y'), where both types of intermittent behavior are observed, $s = 4.4$.

$\varphi_b(s,t)$ is the phase of the signal of breathing at the time scale s and $\varphi_{RR}(2s,t)$ is the phase of the heart rate introduced at the doubled time scale $2s$. The time scale $s = 5$ s corresponds to the frequency of breathing $f_b = 0.2$ Hz and the time scale $2s = 10$ s corresponds to the frequency of slow oscillations in heart rate $f = 0.1$ Hz.

From Fig. 5(a) one can see that the dependence $\Delta\varphi_s(t)$ contains both the regions of synchronous behavior and the phase slips. Unfortunately, due to the objective limitations, we cannot obtain the records, which have enough length for calculation of the laminar phase length distribution and mean length of laminar phases. However, we can apply the rotating plane approach to the recorded data in the same

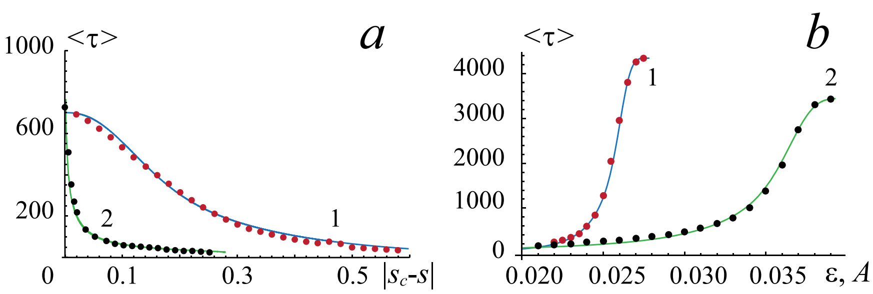

FIG. 4. The mean length $\langle\tau\rangle$ of laminar phases versus the observation time scale s (a) and coupling strength ε (amplitude A of the external signal) (b) for driven Van der Pol oscillator with noise (curve 1) and two coupled Rössler systems (curve 2). The theoretical curves (12) are shown by solid lines and the numerically obtained data are shown by dots. In (a), $s_c = s_h = 8.62$ for curve 1 and $s_c = s_l = 4.99$ for curve 2.

manner as it was done in Fig. 2 to separate from each other the asynchronous phase jumps corresponding to different types of intermittency (Figs. 5(b) and 5(c)). As one can see from Fig. 5(c), the intermittency of intermittencies takes place in the human cardiovascular system. For comparison, we plot also the phase difference and the rotating plane pictures for two coupled Rössler oscillators (10) considered in a similar way to the human cardiovascular system (Figs. 5(a)–5(e)), i.e., the phase difference in this case is calculated as $\Delta\varphi_s(t) = \varphi_1(s,t) - 2\varphi_2(2s,t)$.

In conclusion, we have reported for the first time on the new phenomenon revealed in the temporal behavior of two coupled complex systems at the onset of time scale synchronization regimes at which two different types of intermittency alternate with each other. Such type of intermittency differs greatly from all other types of intermittency known so far. The theoretical dependencies of the mean length of laminar phases on the control parameter values are derived that are in perfect agreement with the numerically obtained data. Since the features of the discovered intermittent process are explicitly deduced at the boundary of the time scale synchronization, we expect that the same phenomenon may occur under certain conditions in a variety of coupled oscillators.

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