ISSN 1062-8738, Bulletin of the Russian Academy of Sciences. Physics, 2011, Vol. 75, No. 12, pp. 1601–1604. © Allerton Press, Inc., 2011.
Original Russian Text © Ye.I. Borovkova, A.S. Karavaev, V.I. Ponomarenko, M.D. Prokhorov, B.P. Bezruchko, 2011, published in Izvestiya Rossiiskoi Akademii Nauk. Seriya Fizicheskaya, 2011, Vol. 75, No. 12, pp. 1704–1708.

Uncovering Frequency Locking for Systems Affected by Chirping

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Abstract—We solve the problem of detecting synchronization epochs for interacting self-oscillating systems from short, nonstationary, noisy time series. To detect epochs of synchronization between the systems we use methods based on estimating the phase coherence coefficient and signal power spectra in a moving window. The efficiency of these methods is demonstrated on simulated time series and experimental physiological signals.

DOI: 10.3103/S1062873811120057

INTRODUCTION

The problem of phase synchronization of oscillations has recently attracted much attention. The detection of synchronization, understood as tuning the phases of self-oscillating systems in the results of their interaction, is dealt with by traditional linear methods of cross-correlation analysis [1], or by such nonlinear methods as calculating the relation of instantaneous frequencies and constructing synchrograms [2], or computing the index of phase synchronization [3], the coefficient of phase coherence [4, 5], and reciprocal information [6].

The field of application for these methods is limited in respect to processing real signals because the detection of intervals of phase synchronization is complicated in this case by real systems being nonstationary, nonlinear, and inevitably subject to noise. In the case of experimental signals, the boundaries of synchronization are eroded, and this substantially complicates diagnostics of synchronization by time series. At present, most of the known methods for measuring the synchronization quality of systems by the signals they produce are oriented toward analyzing stationary signals, and are inadequate for analyzing signals whose basic frequency changes during the time of observation. Under these conditions, specialized approaches designed to analyze signals of an identified class are more efficient.

This work is devoted to comparing methods for diagnosing epochs of the phase synchronization of systems by nonstationary occurrences under the action of external signal of variable frequency.

INVESTIGATED SYSTEMS

A system from [7] was chosen as the first object of investigation. The model system describes the process of regulating vascular tone and cardiac rhythm with a frequency of about 0.1 Hz and has the form of a differential equation with lagging feedback:

$$\varepsilon\dot{x}(t) = -x(t) + f\left(x(t-\tau)\right), \tag{1}$$

where x characterizes the mean aortic pressure; τ is the time of the signal's delay in its propagation about the nervous system, reflecting the time of conductivity of afferent (incoming) and efferent (outgoing) nerves and neurotransmission; parameter ε characterizes the inertia properties of vessels and arterial baroreceptors; and function f describes the nonlinear transformation of signals in central units of the sympathetic nervous system and is

$$f(x) = \frac{c}{\left(1 + a\exp\left[-b\left(x - x^*\right)\right]\right)} - \frac{c}{\left(1 + a\exp\left[b\left(x - x^*\right)\right]\right)}, \tag{2}$$

where the parameters a, b, c and x^* determine the form of the sigmoid nonlinear characteristic.

In order to accommodate the effect of respiration on the cardiovascular system, a modification of model (1) was proposed, in which external action was included:

$$\varepsilon\dot{x}(t) = -x(t) + f\left(x(t-\tau)\right) + A\sin\left(\nu t\right) + \xi, \tag{3}$$

where A and ν are the amplitude and frequency, respectively, of the external force; and ξ is Gaussian white noise. Model (3) parameters $\tau = 3.6$ s, $\varepsilon = 2$ s, $a = 1$, $b = 2$, $c = 2$, and $x^* = 0.5$ were chosen in accordance with the recommendation in [7] that at $A = 0$ ensured a frequency of self-oscillations in the system of about 0.1 Hz. We considered the case of outside action with frequency $f_d = \nu/2\pi$ changing linearly from 0.05 to 0.25 Hz with $A = 1$ and ξ having a zero mean and a mean square deviation (MSD) of 10% that of a series without noise. Our analysis also included experimental data obtained in a natural experiment.

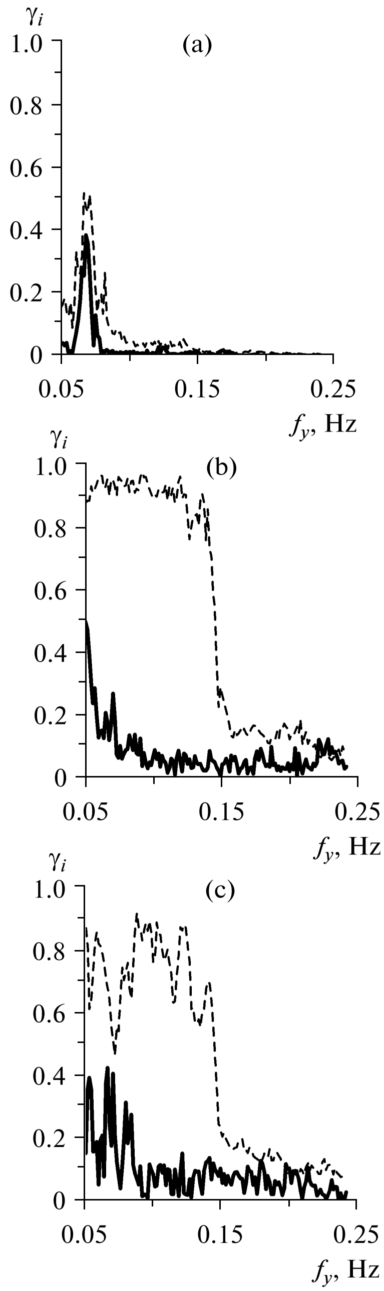

Fig. 1. Coefficient of the phase coherence of (a) a sinusoid with linearly changing frequency and variable x of model (3), (b) R–R epochs, and (c) PPG. A pointwise 95% level of significance is shown by the dotted line.

METHOD

Coefficient of Phase Coherence

To identify the epochs of phase synchronization, we used an approach based on estimating the moving window of the coefficient of phase coherence:

$$\gamma_{n,m}^{12} = \left|\langle \exp(i\varphi_{n,m}^{12}(t))\rangle_t\right| = \sqrt{\langle \cos\varphi_{n,m}^{12}(t)\rangle_t^2 + \langle \sin\varphi_{n,m}^{12}(t)\rangle_t^2}, \quad (4)$$

where $\varphi_{n,m}^{12}$ is the generalized difference of phases, or the relative phase: $\varphi_{n,m}^{12} = n\phi_1 - m\phi_2$, ϕ_1 and ϕ_2 are phases of the first and second signals, n and m are integers, and the angle brackets denote time averaging. We analyzed nonstationary signals, so calculations of the coefficient of phase coherence must be performed in short windows.

To confirm the statistical significance of the results from our investigation, we tested the zero statistical hypothesis in the absence of phase synchronization between interacting systems. The test was performed using surrogate data in whose preparation we used an approach based on mixing the moving windows of one interacting system.

The following procedure was used to estimate the pointwise level of significance: For each ith moving window of the signal M, a number of surrogate windows were selected by enumeration. The value γ_j, $j = 1, ..., M$ was then estimated for each jth surrogate pair of surrogate data. A pointwise 95% level of significance was used in analyzing the results from our investigation by means of such surrogates.

Spectral Analysis

The second approach used to investigate synchronization of the investigated systems was spectral analysis of signals in a moving window. The instantaneous frequencies f_x and f_y of interacting systems were identified from the position of the main peaks in the spectra of signal power. The power spectra were estimated in a moving window with a fixed width. To increase the resolution of the spectra, they were estimated by calculating a periodogram with a square window. The dependency of instantaneous frequency f_x of the oscillations of driven system X on instantaneous frequency f_y of the coercive force was constructed Y.

Parts of the phase synchronization intervals were diagnosed using a diagram constructed on a plane (f_x, f_y) showing areas in which f_x coincides with the value f_y to the accuracy of spectral resolution.

RESULTS AND DISCUSSION

A time series 10000 points long was obtained during the numerical solution of Eq. (3) with the above parameter values and a step of 0.2. The epochs of phase synchronization were then identified using an approach based on estimating the coefficient of phase coherence in the moving window, and via spectral analysis of signals in the moving window.

Figure 1a shows the results from estimating the phase coherence coefficient γ_j in the moving window. The width of window was selected as equal 400 s, and the shift of the window was 10 s. The solid line in Fig. 1a shows the dependency of the coefficient of phase coherence between variable x of model (3) and a sinusoid with a frequency changing linearly from 0.05 to 0.25 Hz on the instantaneous frequency of the sinusoid itself. The dashed line shows the pointwise level of significance. It can be seen that the pointwise 95% level of significance exceeds γ_j for any *i*th window. This is explained by the short length of the window and its nonstationarity, which does not allow its width to be increased.

Figure 2a constructs the dependency of the instantaneous frequency of f_x oscillations in system (3) on instantaneous frequency f_y of harmonic action with linearly changing frequency f from 0.05 to 0.25 Hz. The frequency of oscillations was estimated in short windows 400 s wide. The domain of frequency locking was identified as the domain in which f_x follows f_y. The moment of synchronization loss was taken as the moment when the instantaneous frequency of oscillations of the system was again equal to the characteristic frequency of the oscillations. From Fig. 2a, we can see that at a frequency of outside action of 0.08–0.12 Hz, frequency locking on the order of 1 : 1 was observed.

The data obtained in the natural experiment were also analyzed. We recorded electrocardiograms (ECGs), photoplethysmograms (PPGs), and respiration signals from 11 healthy subjects in the sitting position. During the experiment, the tested subjects inhaled according to the sound signal assigned to the respiration frequency, which changed linearly from 0.05 to 0.25 Hz over 25 minutes. The signals were recorded with a frequency of discretization of 250 Hz and a resolution of 16 bits.

Information on variability of cardiac rhythm was obtained by identifying a sequence of *R–R* epochs from the ECGs, i.e., by constructing a number of time series of epochs T_i between two sequent *R* peaks [8]. Instead of the ordinal number of the *R–R* epoch, we plotted on the abscissa the time of the appearance of *R* peaks $t_k = \sum_{i=1}^{k} T_i$. Interpolating the obtained relationship by cubic splines and choosing from it points at equal time intervals, we obtained an equidistant time series for a beat-to-beat heart rate record. The signals of the beat-to-beat heart rate record and the PPG were oversampled to 5 Hz and filtered by band digital filter with a pass band of 0.05–0.25 Hz.

Figure 1b, c shows the dependencies on the instantaneous frequency of the coefficient of phase coherence γ_j for *R–R* epochs and PPGs, respectively, and for sinusoids with a linearly changing frequency. The pointwise level of significance l exceeds γ_j for any *i*th window. The chosen width of a window was 100 s, and the window shift was 1 s. We may therefore conclude that this approach is inapplicable to identifiying the moments of locking and failure of frequencies by short, nonstationary, noisy time series.

Figre 2b, c shows that the dependency is constructed from the instantaneous frequency of f_x *R–R* epochs and PPGs on the instantaneous frequency f_y of harmonic action with linearly changing frequency. At frequencies of outside action of 0.07–0.21 Hz the locking of frequencies on the order of 1 : 1 between *R–R* epochs and the signal of respiration (Fig. 2b) is observed. The locking of 1 : 1 frequencies between PPGs and the respiration signals is observed at frequencies of outside action of 0.08–0.12 and 0.14–0.19 Hz (Fig. 2c).

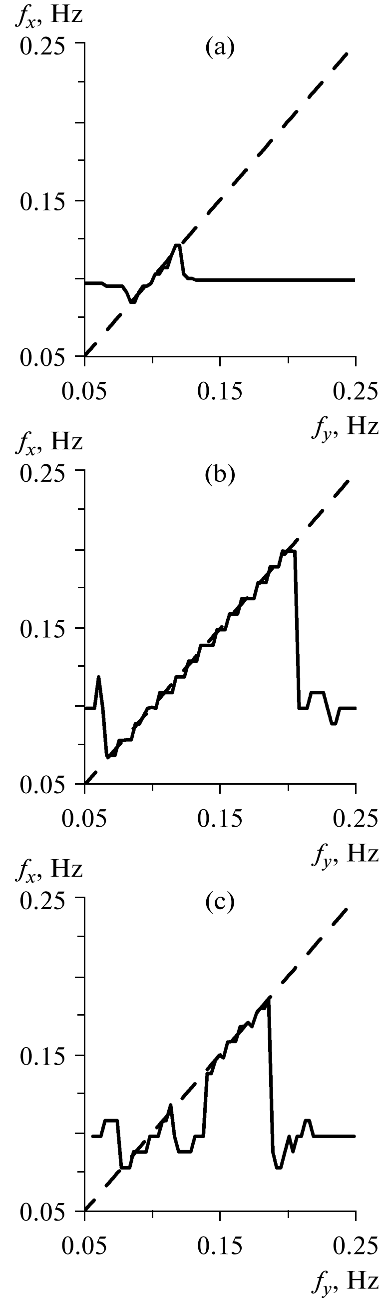

Fig. 2. Dependency of (a) the instantaneous frequency of f_x oscillations in system (3), (b) *R–R* epochs, and (c) PPG on the instantaneous frequency of f_y harmonic action with a frequency changing linearly from 0.05 to 0.25 Hz. The dotted line shows the instantaneous frequency of outside action f_y.

Summarizing the results of our study, we note that the locking range of 0.1 Hz frequencies of oscillations in the *R*–*R* epochs from respiration with smoothly changing frequency proved to be wider than the locking range of 0.1 Hz frequencies of oscillations in PPGs of respiration in 7 subjects (64%), while it was narrower in 4 subjects (36%). The duration of synchronization of *R*–*R* epochs changed from 245 to 1035 s, while the duration of PPG synchronization ranged from 345 to 835 s.

In respiration frequencies close to the self-frequencies of systems for the slow regulation of the cardiac rhythm and blood pressure, we observe locking of the oscillation frequencies of these systems according to respiration as in classical self-excited oscillators upon outside action. The different widths of the synchronization band of the investigated rhythms agree well with the assumption that they are generated by different self-oscillating systems [9].

CONCLUSIONS

We used a method based on the spectral analysis of signals in a moving window to investigate mechanisms of the vegetative regulation of the circulatory system that participate in the formation of 0.1 Hz oscillations in the variability of cardiac rhythm and blood pressure, which are very important for maintaining homeostasis. The mechanism of the onset of these low frequency oscillations is still an object of discussion. A number of researchers believe that 0.1 Hz oscillations in blood pressure and cardiac rhythm are generated in the central section of the system for the vegetative regulation of the cardiovascular system [10, 11]. These oscillations would then reflect the activity of two interacting and self-synchronizing cardiovascular systems. According to another popular viewpoint, these oscillations are mostly a result of the activity of the baroreflex [12, 13]; i.e., the self-oscillating system is only a system for regulating blood pressure.

We showed that 0.1 Hz rhythms of the cardiovascular system can be synchronized with respiration. The different reaction of 0.1 Hz oscillations of the cardiac rhythm and blood pressure to respiration with linearly changing frequency favors the hypothesis as to the existence of two interacting self-oscillating systems with self-frequencies of about 0.1 Hz in the cardiovascular system.

It was shown that the locking of frequencies of 0.1 Hz oscillations of a beat-to-beat heart rate record with smoothly changing frequency was wider than the range of locking 0.1 Hz oscillation frequencies of PPGs by respiration in 7 subjects (64%), and it was narrower in 4 subjects (36%). The different widths of the synchronization bands of the rhythms agree well with the assumption that they are generated by different self-oscillating systems.

Methods for diagnostics of the phase synchronization of interacting self-oscillating systems were compared using short, nonstationary, nonlinear, noisy experimental time series. The efficiency of the approaches was demonstrated during numerical simulations and in analyzing real physiological signals.

It was shown that the method of estimating the coefficient of phase coherence in a moving window is inapplicable because of its high sensitivity to the length of the analyzed time series and their nonstationarity. Direct spectral estimation in a moving window yields results that are qualitatively reproducible in a number of experimental records and in numerical experiments. Our results favor the hypothesis of the self-oscillating character of both systems of regulating cardiac rhythm variability and the system for regulating blood pressure variability, with self-frequencies of about 0.1 Hz.

ACKNOWLEDGMENTS

The work was supported by the Russian Foundation for Basic Research (grant nos. 10-02-00980 and 11-02-00599); by an UMNIK grant; and by the federal target program Developing the Scientific Potential of Higher Education, 2009–2011 (project no. 2.1.1/1738).

REFERENCES

1. Jenkins, G. and Box, G., *Time Series Analysis: Forecasting and Control*, San Francisco: Holden–Day, 1970; Moscow: Mir, 1974, p. 242.
2. Schäfer, C., Rosenblum, M.G., Abel, H.-H., and Kurths, J., *Phys. Rev. E,* 1999, vol. 60, p. 857.
3. Tass, P., Rosenblum, M.G., Weule, J., et al., *Phys. Rev. Lett.*, 1998, vol. 81, p. 3291.
4. Mormann, F., Lehnertz, K., David, P., and Elger, C.E., *Phys. D*, 2000, vol. 144, p. 358.
5. Quiroga, R.Q., Kraskov, A., Kreuz, T., and Grassberger, P., *Phys. Rev. E,* 2002, vol. 65, p. 041903.
6. Kraskov, A., Stögbauer, H., and Grassberger, P., *Phys. Rev. E*, 2004, vol. 69, p. 066138.
7. Ringwood, J.V. and Malpas, S.C., *Am. J. Physiol. Regulat. Integrat. Comp. Physiol.*, 2001, vol. 280, p. R1105.
8. Prokhorov, M.D., Ponomarenko, V.I., Gridnev, V.I., et al., *Phys. Rev. E*, 2003, vol. 68, p. 041913.
9. Kiselev, A.R., Bespyatov, A.B., and Posnenkova, O.M., et al, *Fiziol. Chel.*, 2007, vol. 33, no. 2, p. 69.
10. Malliani, A., Pagani, M., Lombardi, F., and Cerutti, S., *Circulation*, 1991, vol. 84, p. 482.
11. Cooley, R.L., Montano, N., Cogliati, C., et al., *Circulation*, 1998, vol. 98, p. 556.
12. DeBoer, R.W., Karemaker, J.W., and Stracke, J., *Am. J. Physiol. Heart Circ. Physiol.*, 1987, vol. 253, p. H680.
13. Bernardi, L., Leuzzi, S., Radaelli, A., et al., *Clin. Sci.*, 1994, vol. 87, p. 649.