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Detection of couplings in ensembles of stochastic oscillators

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The problem of detection and quantitative characterization of directional couplings in an ensemble of noisy oscillators from a time series is addressed. We suggest estimators for the strengths of couplings which are based on modeling the observed oscillations with a set of stochastic phase oscillators and easily interpreted from a physical viewpoint. Moreover, we present an analytic formula for a statistical significance level allowing to reveal an architecture of couplings reliably from a relatively short time series. The technique applies to weakly coupled nonsynchronized oscillators. It is introduced for oscillators with close basic frequencies but can be readily generalized to the case of arbitrary frequencies. Efficiency of the technique is demonstrated in numerical experiments.

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I. INTRODUCTION

Couplings between elements determine to a significant extent the entire dynamics in ensembles of oscillatory systems. Multiple works are devoted to spatial and temporal patterns arising under various coupling architectures and strengths, in particular, to different kinds of synchronization [1–6]. In analysis of experimental data an inverse problem is ubiquitous: to restore couplings from a time series. Apart from diagnostics of synchronization [7–14], much attention is paid during the last decade to the detection of *directional* couplings or *causal* influences [15–19] and to the distinction between "direct" and "mediated" couplings [20]. The latter problems are important in different fields of natural science including neurophysiology [21–28], climatology [29–31], electrochemistry [32,33], etc.

To reveal causal influences between two processes, one uses such concepts and approaches as Granger causality [19,34], information-theoretic characteristics [35–38], state space analysis [39–41], double-wavelet analysis [42,43], and phase dynamics modeling [15,16,18]. The latter approach relies on ideas from the nonlinear theory of oscillations and appears the most sensitive one for nonlinear systems with a relatively stable oscillation period [44,45]. Estimators of couplings in ensembles of oscillators based on phase dynamics modeling have been suggested in [33,46] where the problems are formulated and solved for *deterministic* processes. In Ref. [46] one requires close individual frequencies of oscillators and possibility to manipulate them. Coupling strengths are determined from a set of time series corresponding to different *strictly synchronous* regimes. In Ref. [33] estimators are calculated from a single time series but a coupling architecture must be simple and *a priori* known (e.g., all-to-all) while generalizations are practically difficult since the number of estimated quantities should remain small.

In this paper, we suggest a technique to determine an architecture and strengths of couplings which is also based on phase dynamics modeling but applies to more general situations:

(i) Oscillators may be stochastic in contrast to [33,46] that is often crucial for practical applications. The generalization is achieved via incorporation of noise terms into model equations and their proper consideration in derivation of coupling estimators.

(ii) Couplings are estimated from a single time series of a moderate length that is important in a usual case of data deficit or nonstationarity.

(iii) Coupling estimators are supplied with an analytic significance level. It allows reliable conclusions about coupling presence without time-consuming surrogate data analysis and makes processing of large amounts of data much easier. The latter is highly required in biomedical applications.

(iv) The estimators assess direct influences, i.e., not mediated by other observables. Thereby, one reveals a coupling architecture from data.

(v) Closeness of individual frequencies is not compulsory even though it simplifies the problem.

(vi) Numerical values of coupling estimators are readily interpreted from a physical viewpoint.

Below, we discuss theoretical coupling characteristics in Sec. II and their estimators in Sec. III. Performance of the technique is illustrated with numerical simulations in Sec. IV. Summary is given in Sec. V.

II. CHARACTERISTICS OF COUPLING

A general idea of the approach is that "coupling strength" shows how strongly future evolution of an oscillator phase depends on the current values of the other oscillator phases. It continues the line of research [15,18]. The formalism is described below.

Dynamics of an ensemble of weakly coupled (deterministic) limit-cycle oscillators with close natural frequencies can be accurately described with a set of ordinary differential equations involving only resonant interaction terms [47],

$$\frac{d\phi_k}{dt} = \omega_k + \sum_{j=1(j\neq k)}^{M} K_{k,j}\sin(\phi_j - \phi_k), \qquad (1)$$

where $k=1,\ldots,M$, ϕ_k are phases of the oscillators, ω_k are their individual frequencies, and $K_{k,j}$ characterize coupling strengths. Weak coupling condition implies $|K_{k,j}| \ll \omega_k$ for all

k. Since natural frequencies are close, a characteristic oscillation period for all the oscillators can be defined as $T = 2\pi/\bar{\omega}$, where $\bar{\omega} = (1/M)\sum_{k=1}^{M}\omega_k$.

If individual dynamics of each oscillator are not adequately represented by a limit cycle, e.g., its phase can be considerably affected by an external noise or a time-varying amplitude, Eq. (1) is no longer an appropriate model. However, if the disturbances mentioned are weak enough, then model (1) can be slightly generalized through incorporation of noise terms necessary to describe the deviations of the phase evolution from deterministic dynamics (1) (see, e.g., [48]). Thereby, one comes to the description of an ensemble dynamics with a set of stochastic differential equations,

$$\frac{d\phi_k}{dt} = \omega_k + \sum_{j=1(j\neq k)}^{M} K_{k,j}\sin(\phi_j - \phi_k) + \xi_k(t), \qquad (2)$$

where $k=1,\ldots,M$ and ξ_k are independent noise sources whose intensity is small as compared to the contribution of the other terms at time scales of interest. Thus, one typically studies phase variations over time scales of T and longer rather than very fast fluctuations. Under such consideration, if autocorrelations of ξ_k decay down to zero for time lags much less than T, then ξ_k can be adequately described as Gaussian white noises with autocorrelation functions (ACFs) $\langle\xi_k(t)\xi_k(t')\rangle = \sigma_{\xi_k}^2\delta(t-t')$, where angular brackets denote expectation and $\sigma_{\xi_k}^2$ characterizes noise intensity.

Under the conditions of weak noises and couplings, one can readily integrate Eq. (2) over a finite time interval $(t, t+\tau)$ and get difference model equations,

$$\Delta\phi_k(t) = F_k(\phi_1(t),\ldots,\phi_M(t)) + \varepsilon_k(t), \qquad (3)$$

where $k=1,\ldots,M$, $\Delta\phi_k(t) \equiv \phi_k(t+\tau) - \phi_k(t)$, τ is a fixed time interval much greater than autocorrelation times of all ξ_k and not much greater than T, e.g., $\tau = T$, $\varepsilon_k(t) \approx \int_t^{t+\tau}\xi_k(t')dt'$ are independent Gaussian zero-mean noises with variances $\sigma_{\varepsilon_k}^2$ and ACFs linearly decaying from $\sigma_{\varepsilon_k}^2$ to 0 over an interval of time lags $(0,\tau)$ and equal to zero for greater lags [18], and functions F_k read

$$F_k(\phi_1,\ldots,\phi_M) = \alpha_{k,0} + \sum_{j=1(j\neq k)}^{M}(\alpha_{k,j}\cos(\phi_j-\phi_k) + \beta_{k,j}\sin(\phi_j-\phi_k)). \qquad (4)$$

The functions F_k are obtained via integration of all the terms in the right-hand side of Eq. (2) except for ξ_k. At first approximation, one can integrate them even analytically since under the above weakness conditions one can take $\phi_k(t') \approx \phi_k(t) + \omega_k(t'-t)$ for $t < t' < t+\tau$ under the integral sign. Equation (3) with function (4) constitutes a basic phase dynamics model serving us further to define coupling strengths.

The left-hand side of Eq. (3) is a phase increment $\Delta\phi_k$ for the kth oscillator over an interval τ. One can use any value of τ from the range described above since it is a free parameter of the technique [49]. Further, we always set $\tau = T$ as in [15,18] for definiteness (see also Sec. III). Then, $\langle\Delta\phi_k\rangle = \alpha_{k,0} \approx 2\pi$ and $\Delta\phi_k$ fluctuates about this mean value due to noise ε_k and influences from other oscillators. For weak couplings, probability distribution of a wrapped phase vector $(\phi_1,\ldots,\phi_M)$ in an M-dimensional cube $[0,2\pi]^M$ is close to the uniform one. Hence, function terms in the right-hand side of Eq. (4) are mutually orthogonal: $\langle\cos(\phi_n-\phi_k)\cos(\phi_m-\phi_k)\rangle = 0$, $m\neq n$, etc. Therefore, the variance of $\Delta\phi_k$ reads

$$\sigma_{\Delta\phi_k}^2 = \sigma_{\varepsilon_k}^2 + \sum_{j=1,j\neq k}^{M} c_{j\to k}, \qquad (5)$$

where $k=1,\ldots,M$, and

$$c_{j\to k} = \tfrac{1}{2}[\alpha_{k,j}^2 + \beta_{k,j}^2]. \qquad (6)$$

We call the quantity $c_{j\to k}$ *strength of the influence* $j\to k$, i.e., from the jth oscillator to the kth one.

Thus, according to Eqs. (5) and (6) variations in a phase increment $\Delta\phi_k$ (in essence, intensity of frequency modulation) are represented as a sum of M independent factors: its own noise and other oscillators of an ensemble. It gives a clear physical (or, at least, "dynamical") sense to the introduced quantitative characteristics of coupling. We note that $c_{j\to k}$ is similar to the quantity $\langle(\partial F_k/\partial\phi_j)^2\rangle$ used in [15,18] for the case of two oscillators. However, both quantities differ if higher-order terms are incorporated into Eq. (4) as described in Sec. III when $\langle(\partial F_k/\partial\phi_j)^2\rangle$ does not allow such a clear interpretation as $c_{j\to k}$.

III. ESTIMATION FROM A TIME SERIES

In practice, one has only time series $\{x_k(t), t=n\Delta t, n=1,2,\ldots,N'\}$, $k=1,\ldots,M$, where x_k are observables and Δt is a sampling interval, rather than Eqs. (3) and (4). According to the suggested approach, one assumes that phase dynamics of an investigated ensemble of M oscillators is adequately described with a set of stochastic difference equations [Eq. (3)] with function (4). Since coefficients in Eq. (4) are unknown and direct use of formula (6) is impossible, one should compute the phases $\{\phi_k(t), t=n\Delta t, n=1,2,\ldots,N\}$ and *estimate* coupling strengths $c_{j\to k}$.

Here, we do not go into detail of phase computation. This is a separate problem which is often not easy and considered in multiple works. We assume that phases of the oscillators are well defined and can be readily computed with established techniques (e.g., [50,51]). To estimate coupling strengths, one can first estimate coefficients $\alpha_{k,j}$, $\beta_{k,j}$ by minimizing a mean squared prediction error,

$$S(\mathbf{a}_k) = \frac{\sum_{n=1}^{N-\tau/\Delta t}\{\Delta\phi_k(n\Delta t) - F_k[\phi(n\Delta t),\mathbf{a}_k]\}^2}{N-\tau/\Delta t}, \qquad (7)$$

where $\phi = \{\phi_k\}$ is a vector of phases and $\mathbf{a}_k = \{\alpha_{k,j},\beta_{k,j}\}$ is a vector of the kth polynomial coefficients. Thus, their estimators $\hat{\alpha}_{k,j},\hat{\beta}_{k,j}$ are found as $\hat{\mathbf{a}}_k = \arg\min_{\mathbf{a}_k} S(\mathbf{a}_k)$. It would be natural to take $\hat{c}_{j\to k} = \frac{1}{2}[\hat{\alpha}_{k,j}^2 + \hat{\beta}_{k,j}^2]$ as an estimator of $c_{j\to k}$. If a time series is *very long* then $\hat{\alpha}_{k,j},\hat{\beta}_{k,j}$ almost coincide with the "true" values $\alpha_{k,j}$, $\beta_{k,j}$ and, hence, $\hat{c}_{j\to k} = c_{j\to k}$. However, in practice a time series is often of a moderate length. Therefore, $\hat{\alpha}_{k,j},\hat{\beta}_{k,j}$ are not equal to $\alpha_{k,j},\beta_{k,j}$ and one must indicate expected errors.

A central question is: Under what conditions can one infer the presence of an influence $j \to k$, i.e., state that $c_{j\to k} > 0$? Such an inference cannot be made from the inequality $\hat{c}_{j\to k} > 0$ which holds true almost always even for $c_{j\to k}=0$ since $\hat{c}_{j\to k}$ is a sum of squares of two estimators which are almost always nonzero at least due to random fluctuations.

To find a distribution law of $\hat{c}_{j\to k}$ and a significance level for the conclusion $c_{j\to k} > 0$, we note that under the null hypothesis $c_{j\to k}=0$ the quantities $\hat{\alpha}_{k,j}, \hat{\beta}_{k,j}$ are independent and identically distributed according to Gaussian law with zero mean and some variance $\sigma^2_{\hat{\alpha}_{k,j}}$. Hence, a quantity $\hat{\chi}^2_{j\to k} = 2\hat{c}_{j\to k}/\sigma^2_{\hat{\alpha}_{k,j}}$ is distributed according to χ^2_2 law. Its distribution function $\Phi_2(x)$ is tabulated (see, e.g., [52]). Let us denote $\chi^2_{2,(1-p)}$ such a number that $\Phi_2(\chi^2_{2,(1-p)}) = 1-p$. If $\hat{\chi}^2_{j\to k} > \chi^2_{2,(1-p)}$, the null hypothesis is rejected at a significance level p, i.e., with error probability not greater than p. In particular, $p=0.05$ is often used in practice to provide sufficiently high reliability. The value of $\sigma^2_{\hat{\alpha}_{k,j}}$ is *a priori* unknown but one can substitute its estimate $\hat{\sigma}^2_{\hat{\alpha}_{k,j}}$ derived in [18]:

$$\hat{\sigma}^2_{\hat{\alpha}_{k,j}} = 2\hat{\sigma}^2_{\varepsilon_k}/N\{1 + 2\sum_{l=1}^{\tau/\Delta t}[1 - l/(\tau/\Delta t)]\cos[(\hat{\alpha}_{k,0} + \hat{\alpha}_{j,0}) \times l/(\tau/\Delta t)]\exp[-(\hat{\sigma}^2_{\varepsilon_k} + \hat{\sigma}^2_{\varepsilon_j})l/(2\tau/\Delta t)]\},$$

where $\hat{\sigma}^2_{\varepsilon_k} = \min_{\mathbf{a}_k} S(\mathbf{a}_k)$. Thus, a significance level at which one detects the influence $j\to k$ is assessed as $\hat{p}_{j\to k} = \Phi_2^{-1}(2\hat{c}_{j\to k}/\hat{\sigma}^2_{\hat{\alpha}_{k,j}})$.

To derive the formula for $\hat{\sigma}^2_{\hat{\alpha}_{k,j}}$, one relies on whiteness of ξ_k or, equivalently, Gaussianity of ε_k and linear decay of their ACFs over an interval $(0,\tau)$ [18]. This is the only point where the assumed properties of the noises come into play but it is important since otherwise one could not assess expected estimation errors. However, Gaussianity of ε_k is not compulsory and even the required ACF behavior can be moderately violated as shown in [53] that exhibits certain robustness of the technique.

In practice, it may be useful to compare the strengths of the influences $j\to k$ for different j and k to each other. However, the estimator $\hat{c}_{j\to k}$ is biased; its bias can be derived similarly to [18] and equals to $\sigma^2_{\hat{\alpha}_{k,j}}$. The biases in $\hat{c}_{j\to k}$ may differ strongly for different pairs (j,k) that mislead if one compares the values of $\hat{c}_{j\to k}$ to each other. Hence, an unbiased estimator is desirable which is given analogously to [18] by

$$\hat{C}_{j\to k} = \tfrac{1}{2}[\hat{\alpha}^2_{k,j} + \hat{\beta}^2_{k,j} - 2\hat{\sigma}^2_{\hat{\alpha}_{k,j}}]. \tag{8}$$

Despite our formulas strictly apply asymptotically, they work well for time series of a moderate length as illustrated in Sec. IV. However, some empirical criteria of applicability related to the conditions of well-defined phases and weak couplings and noises should be checked to assure reliability of the estimation results:

(i) Condition of well-defined phases can be formulated theoretically as narrow-band power spectra of the signals x_k or as weak variation in $\Delta\phi_k$, i.e., $\sigma_{\varepsilon_k} \ll \alpha_{k,0}$ for all k. In our experience, it is practically sufficient to require $\hat{\sigma}_{\varepsilon_k} < 0.2\hat{\alpha}_{k,0}$ for all k that is fulfilled in all the examples below.

(ii) Condition of weak coupling theoretically reads $|\alpha_{k,j}|, |\beta_{k,j}| \ll \alpha_{k,0}$ for all $j>0$ and all k. In practice it appears more important to check a result of such weakness, i.e., to check whether weak interdependence between simultaneous values of ϕ_j and ϕ_k holds true to assure approximately uniform distribution of the vector ϕ in an M-dimensional cube. More precisely, it is sufficient to meet the less strict condition of mutually orthogonal function terms in the right-hand side of Eq. (4). Theoretically, the orthogonality holds true if the phase synchronization index $\rho_{j,k} = |\langle\exp\{i[\phi_j(t) - \phi_k(t)]\}\rangle| = 0$ for all j,k where angular brackets denote expectation. In practice, we check whether the condition is approximately fulfilled by estimating $\rho_{j,k}$ as $\hat{\rho}_{j,k} = |(1/N)\sum_{n=1}^{N}\exp\{i[\phi_j(n\Delta t) - \phi_k(n\Delta t)]\}|$ [7] and requiring $\hat{\rho}_{j,k} < \rho_c$ for all j,k. If $\hat{\rho}_{j,k} > \rho_c$, then a time series is not used for coupling estimation. The threshold ρ_c is found below in numerical experiments with different oscillators and $\rho_c = 0.45$ appears to suffice in all cases.

We note that if couplings between oscillators are very strong, the value of $\hat{\rho}_{j,k}$ is very high for almost any time realization. In such a case, the condition $\hat{\rho}_{j,k} < \rho_c$ is almost never fulfilled that leads to a very low rate of correct positives. Hence, despite the technique gives no false positives, it gets practically useless since it is no longer sensitive to the presence of couplings. However, revealing directionality of strong coupling is known to be a principally difficult problem (e.g. [44]).

(iii) Condition of the ACF decay over an interval of time lags $(0,\tau)$ for the noises ε_k can be checked by estimation of those ACFs from the residual errors of the fitted model (3). One can try to increase the interval τ until the ACF decay condition is fulfilled. Typically, $\tau=T$ is a good choice which is used in all examples below, but in general it does not automatically assure fulfilling of this condition. Still, the coupling estimators may appear applicable even if the condition is moderately violated [53].

(iv) Another is the condition of a sufficient length of a time series. The number of coefficients in each function (4) should be much less than a number of nonoverlapping intervals of the duration τ in a time series: $2M-1 \ll N\Delta t/\tau$. Since a usual choice is $\tau=T$ and, moreover, consideration of smaller τ is not reasonable (it would mean description of fast phase fluctuations which depend on concrete technical way of the phase determination while variations in the value of the oscillation period are usually much better defined), it is convenient to formulate the requirement in terms of a mean oscillation period T: $2M-1 \ll N\Delta t/T$. Minimal time series lengths for different M are reported below as a result of numerical simulations.

(v) Condition of close basic frequencies ω_k can be checked directly from the estimation results as $|\hat{\alpha}_{k,0} - \hat{\alpha}_{j,0}| \ll \hat{\alpha}_{j,0}$ for all j,k. However, the technique appears applicable even if this condition is strongly violated. In case of essentially different frequencies the technique may appear insensitive if existing couplings require a description with higher-order terms in Eq. (4), but it does not lead to spurious coupling detection. A similar situation takes place if nonresonant interaction terms to be described with higher-order terms in Eq. (4) are considerable.

However, it is straightforward to make the technique more sensitive in cases of different frequencies ω_k or nonresonant interaction terms. One has just to incorporate higher-order terms $\cos(m\phi_j - n\phi_k)$ and $\sin(m\phi_j - n\phi_k)$ with $m > 1$ and/or $n > 1$ into Eq. (4) that would correspond to more general coupling functions in Eq. (2). Then, there would be L terms under the summation sign in Eq. (4) instead of the two terms. Coupling strengths would be defined as in Eq. (6) but via L coefficients. Coupling estimators would be distributed according to χ_L^2 law rather than χ_2^2 law. Individual nonlinearities could be accounted for by the terms $\cos(m\phi_k)$ and $\sin(m\phi_k)$ with $m > 0$ or even eliminated by converting to invariant phases [54]. The weak coupling condition would imply low values of the generalized index $|\langle \exp\{i[m\phi_j(t) - n\phi_k(t)]\}\rangle|$ for all the orders (m,n) used in Eq. (4) that is closer to a strict requirement of the uniformity of the phase vector distribution over the M-dimensional cube.

Below, we use only the first-order terms in Eq. (4) for the sake of brevity. A generalized technique is considered in detail elsewhere.

Thus, the theoretical assumptions underlying our coupling estimators are not very restricting. Moreover, with the above empirical criteria, the approach appears suitable for moderate violation of the theoretical conditions, in particular, for moderately strong couplings and relatively short time series as illustrated below.

IV. NUMERICAL SIMULATIONS

We consider several exemplary systems at various coupling strengths and architectures, time series lengths N, and ensemble sizes M. In all the examples $\bar{\omega}=1$. For the experiments with phase oscillators (Secs. IV A and IV B) sampling interval is $\Delta t=0.6$, i.e., approximately 10 data points per a mean oscillation period $T=2\pi$. For the other examples (Sec. IV C) where one needs to compute phases from observed signals, sampling interval should be small enough to avoid distortions. At least 20 data points per a basic period are recommended in [51]; therefore, we use $\Delta t=0.3$ for those examples. As discussed above, we set $\tau \approx T$, i.e., $\tau=10\Delta t$ or $\tau=20\Delta t$, respectively. Under fixed number of oscillation periods in a time series, the estimation results almost do not depend on the sampling interval Δt, which can be understood as follows. Decrease in Δt and, hence, increase in N add new data points $\phi_k(t')$ "between" existing data points $\phi_k(t)$. Those new data points carry information about fast fluctuations of phases over time intervals of the order of Δt. Since we are interested in the analysis at time scales $\tau=T$, such detailed information is just averaged out when phase increments $\Delta\phi_k(t)$ in Eq. (3) are computed. Hence, only the number of oscillations periods in a time series is important. Therefore, we present all the results for the above fixed values of Δt.

We use the Euler technique to integrate stochastic differential equations and the fourth-order Runge-Kutta technique for ordinary differential equations. Integration step is 0.001. For each exemplary system and each set of its parameters, we generate a large collection of its time realizations to assess statistically the performance of the suggested technique.

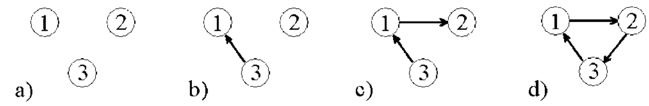

FIG. 1. Considered coupling architectures in a small ensemble of phase oscillators [Eq. (2)].

The number of time series in such a collection is denoted N_s. We use $N_s=1000$ in Sec. IV A and $N_s=100$ in Secs. IV B and IV C. From each time series in a collection, we estimate couplings for each pair of oscillators j,k and either infer the presence of the influence $j \to k$ (a positive conclusion) or not (a negative one) at a given significance level p. Positives are correct if the influence $j \to k$ indeed exists and false otherwise. The rate of positives is denoted $\nu_{j\to k}$ [55].

In all tests, we check two points. First, the suggested estimators are regarded applicable if the rate of false positives $\nu_{j\to k}$ does not exceed p up to acceptable fluctuations due to finiteness of N_s [55] for all j,k such that $c_{j\to k}=0$. Most often, we use $p=0.05$. Second, it is desirable for the technique to be sensitive, i.e., to give a considerable rate of correct positives.

A. Three phase oscillators

The first example corresponds to the simplest case (a small ensemble of phase oscillators) and demonstrates applicability of the suggested estimators for time series of a *moderate* length. A system under investigation is set (2) with $M=3$, $\omega_1=1.1$, $\omega_2=1.0$, $\omega_3=0.9$, and $\sigma_{\xi_k}^2=0.04$ for all k. Throughout this section, a time series length is fixed to be $N=1000$, i.e., 100 oscillation periods in a time series. The coefficients $K_{j\to k}$ determine a coupling architecture in an ensemble. We have considered four versions:

(1) uncoupled oscillators, all $K_{j\to k}=0$ [Fig. 1(a)];

(2) only $K_{3\to 1}=0.05$, others are zero [Fig. 1(b)];

(3) $K_{3\to 1}=0.05$ and $K_{1\to 2}=0.025$ [Fig. 1(c)]; and

(4) "ring" coupling, $K_{3\to 1}=0.05$, $K_{1\to 2}=0.025$, and $K_{2\to 3}=0.075$ [Fig. 1(d)].

For uncoupled oscillators [Fig. 1(a)], all positives are false so that all rates $\nu_{j\to k}$ should not exceed a given significance level p. In Fig. 2(a) the plots $\nu_{j\to k}(p)$ for two pairs j,k are shown. The condition $\nu_{j\to k}(p) \le p$ is fulfilled so that the suggested technique performs properly for any p. When couplings are introduced according to any of the architectures shown in Figs. 1(b)–1(d), false positive rates approximately equal to p [Figs. 2(b)–2(d)] within an acceptable range of fluctuations [55]. Further, a correct detection rate in case of Fig. 1(b) is high: $\nu_{3\to 1}=0.9$ at $p=0.05$ [Fig. 2(b)]. It decreases with decreasing p which is a payment for a smaller probability of false positives. However, $\nu_{3\to 1}$ is still high (about 0.6) even at $p=0.001$. Other correct detection rates for the ensembles shown in Figs. 1(c) and 1(d) behave in a similar way [Figs. 2(c) and 2(d)]. As expected, the detection rate $\nu_{j\to k}$ rises with $K_{j\to k}$, e.g., $\nu_{1\to 2}=0.5$, $\nu_{3\to 1}=0.9$, and $\nu_{2\to 3}=1.0$ at $p=0.05$ in Fig. 2(d). Note that in the example of Fig. 1(c) an oscillator 3 drives an oscillator 2 via an oscillator 1, i.e., through a mediated coupling. However, the influence $3\to 2$ is not detected by the suggested estimators. It illustrates that they assess only direct couplings.

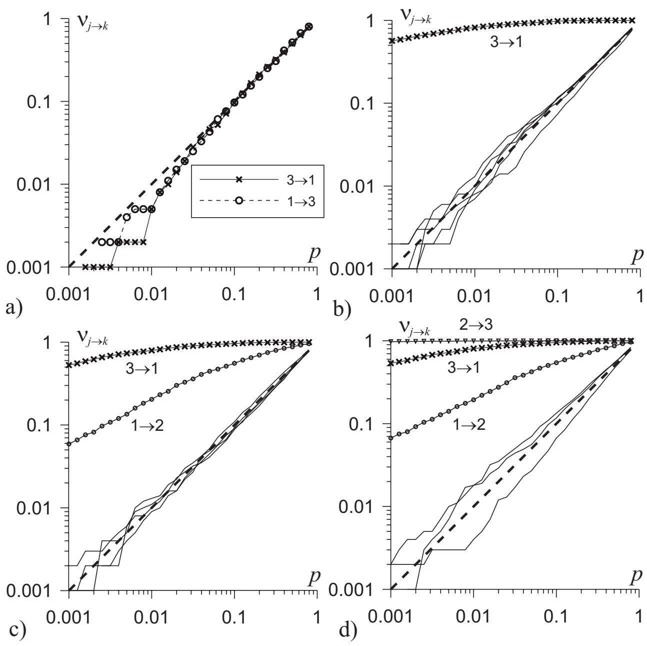

FIG. 2. Rates of positives for a small ensemble (2) consisting of three phase oscillators versus a specified significance level p. Panels (a)–(d) correspond to panels (a)–(d) in Fig. 1. (a) All positives are false. For the sake of convenience, only two of the six plots are shown, all the rest are analogous. (b) Only detections of the influence $3\to1$ are correct. Their rate is high. (c) Detections of the influences $3\to1$ and $1\to2$ are correct. (d) Detections of the influences $3\to1$, $1\to2$, and $2\to3$ are correct. In all cases, a false positive rate fluctuates about the value of p within acceptable limits [55] that implies a proper performance of the suggested estimators.

False positive rates may exceed a given level p under increase in coupling coefficients or decrease in noise level and frequency mismatch. Figure 3(a) illustrates variations in $K_{3\to1}$ for the case of Fig. 1(b): error probability does not exceed $p=0.05$ up to $K_{3\to1}=0.14$ but starts to rise with further increase in $K_{3\to1}$. The rise of the error rate is determined by a considerable violation of the weak coupling condition. With a criterion $\hat{\rho}_{j,k}<\rho_c$, the rate of false positives does not exceed an acceptable value (the dashed line) in the entire range of $K_{3\to1}$ for $\rho_c=0.45$. This threshold is close to the values reported in [56] for the case of two oscillators. With analogous tests, we have observed that $\rho_c=0.45$ is sufficient to control false positive rates in all examples considered below (under an additional requirement of a sufficient time series length N which is analyzed in Sec. IV B). Sensitivity of the technique after addition of the criterion $\hat{\rho}_{j,k}<\rho_c$ remains high: the rate of correct positives is the same as without the criterion at small $K_{3\to1}$ [Fig. 3(b)]. The sensitivity gets worse for big $K_{3\to1}$ but still equals unity for a wide range $0.07\le K_{3\to1}\le 0.13$.

Coupling strength estimator $\hat{C}_{3\to1}$ on average depends on $K_{3\to1}$ as a quadratic function in the entire range of reliable coupling detection [Fig. 3(c)]. It is expected since $c_{j\to k}$ is proportional to the sum of squared coefficients [Eq. (6)]. Thus, at small τ Eq. (3) would almost coincide with the Euler scheme so that one can derive analytically $\langle\hat{C}_{3\to1}\rangle=(\tau^2/2)K_{3\to1}^2$. The coefficient in the formula differs from

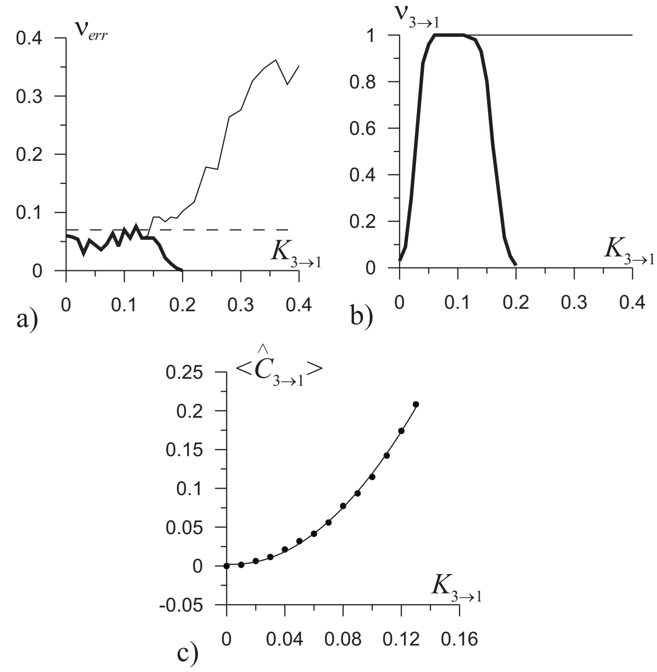

FIG. 3. Coupling estimation for the ensemble of three phase oscillators [Fig. 1(b)] under variation in $K_{3\to1}$: (a) a total false positive rate at $p=0.05$ without control of $\hat{\rho}_{j,k}$ (thin line) and with $\hat{\rho}_{j,k}<0.45$ (thick line), an acceptable error level [55] is shown with dashed line; (b) detection rates for the influence $3\to1$ at $p=0.05$ without control of $\hat{\rho}_{j,k}$ (thin line) and with $\hat{\rho}_{j,k}<0.45$ (thick line); and (c) mean values of $\hat{C}_{3\to1}$ (circles) with an approximating quadratic parabola (solid line).

$\tau^2/2$ at greater τ, for instance, $\langle\hat{C}_{3\to1}\rangle\approx 12.8K_{3\to1}^2$ in Fig. 3(c) where $\tau=6$. However, quadratic character of the dependence is preserved.

Effects of decrease in the noise level, frequency mismatch, and a time series length are analyzed in the same way (not shown). The criterion $\rho_{j,k}<\rho_c$ also helps us to control false positives with the same ρ_c. Only time series length variations exhibit a peculiarity: N should not be less than a certain minimal value for a given ensemble size M. This is because the above formulas for the estimators, e.g., Eq. (5), are asymptotic so that they require significant number of nonoverlapping intervals of the width τ in a time series. Minimal values of N versus an ensemble size M are reported in Sec. IV B.

B. Larger ensembles of phase oscillators

To study an ensemble size effect, we use system (2) with different M. First, we consider an ensemble of $M=10$ oscillators with a simple ring architecture [Fig. 4(a)], i.e., $K_{j\to k}\neq 0$ only for $j=k-1$, $k>1$ or $j=M$, $k=1$. These coupling coefficients $K_{j\to k}$ rise with k and there is a certain frequency mismatch: $K_{j\to k}=0.03+0.003k$, $\omega_k=1.275-0.05k$, and $k=1,\ldots,10$. Noise intensities are the same $\sigma_{\xi_k}^2=0.04$.

We perform estimation from time series of the length $N=10\,000$ (1000 basic periods) and other parameters the same as in Sec. IV A. Estimation results for a single time series are

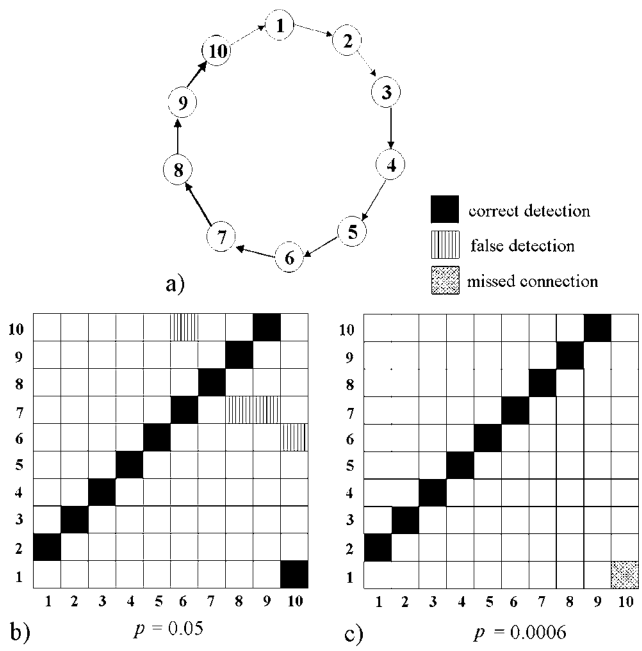

FIG. 4. Coupling estimation for an ensemble of 10 phase oscillators with ring couplings [Eq. (2)] from a single time series: (a) coupling architecture, arrows show existing influences, their thickness indicate the value of a corresponding coefficient $K_{j\to k}$; [(b) and (c)] diagrams for coupling estimation results show detected existing couplings (black squares), missed existing couplings (double shading), false positives (vertical shading), and undetected nonexisting couplings (white squares) at a given significance level p (see the text for additional explanations).

shown in Figs. 4(b) and 4(c). The diagrams illustrate couplings which are detected correctly or spuriously. A square with a horizontal coordinate j and a vertical coordinate k denotes the influence $j\to k$, except for the squares on the diagonal which do not carry any information. All ten existing couplings are detected at the significance level $p=0.05$ (black squares). Four couplings are detected spuriously (vertically shaded squares). However, this rate of errors lies within acceptable range at the given p. Indeed, there are 90 possible couplings in the ensemble; 80 of them are absent, i.e., correspond to zero coefficients $K_{j\to k}$. At the given significance level, one expects 5% of false positives on average, i.e., four of 80 couplings. Estimation from other time series gives similar pictures with slightly fluctuating rate of false positives lying within acceptable limits [55].

To decrease the number of false positives, one should specify smaller p. As illustrated in Fig. 4(c) for the same time series, the choice of $p=0.0006$ provides zero false positives. It is expected since on average one should get $0.0006\times 80=0.048$ false positives at a single diagram, i.e., probability of at least one spuriously detected coupling is less than 0.05. At that, probability to miss existing connections rises. Indeed, the weakest of the existing influences $10\to 1$ is not detected at $p=0.0006$ (a double shaded square). It can be detected only if the time series length in terms of the number of basic periods is increased. Such an improvement in sensitivity is discussed in [44]. Estimates of coupling strengths $\hat{C}_{j\to k}$ for the same time series depend on k in approximately quadratic manner (Fig. 5) since the coupling coefficient $K_{j\to k}$ rises linearly with k. The dependence $\hat{C}_{j\to k}(k)$ is fluctuating since it represents estimates for a single time series rather than averaged characteristics as in Fig. 3(c).

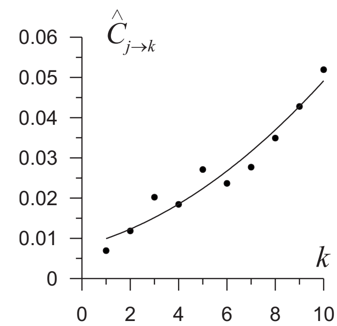

FIG. 5. Coupling estimates $\hat{C}_{j\to k}$ (circles) for an ensemble of 10 phase oscillators [Fig. 4(a)] versus an oscillator number. An approximating quadratic parabola is shown with the solid line.

Applicability if the technique in the sense of controlled rate of false positives is maintained for a time series of the length of 300 basic periods at the expense of the lower sensitivity. Thus, for such a shorter segment of the time series analyzed above, there are two missed connections at $p=0.05$ and four ones at $p=0.0006$ as compared to Figs. 4(b) and 4(c).

Performance of the technique is further illustrated for a more complicated coupling architecture generated randomly [Fig. 6(a)]. There are 20 nonzero coupling coefficients. Bidirectional coupling (for the pair of oscillators 4 and 6) is present along with unidirectional ones. All nonzero coupling coefficients are equal to 0.025. Estimation results for a single time series of the length $N=3000$ (300 basic periods) are shown in Figs. 6(b) and 6(c) analogously to the previous example. Existing couplings of 18 out of 20 are detected at $p=0.05$. Two couplings are detected spuriously that are an acceptable rate at the given p. Smaller value of $p=0.005$ provides absence of spurious detections but the number of missed existing connections rises (six of 20). Again, one should increase the time series length to detect them. The results presented in Fig. 6 are typical as observed from the analysis of an ensemble of time series (not shown in more detail). As distinct from the previous example, not only the number of missed connections but also their locations fluctuate from one time series to another one. The reason is that all couplings are equally strong and which of them are not detected from a given time series is determined by concrete realizations of noises ξ_k corresponding to that time series.

We note that for both examples of ensembles consisting of $M=10$ oscillators, the suggested estimators appear applicable only for a time series length at least of 300 basic periods. To study the requirements on the time series length for different M in detail, we analyze ensembles [Eq. (2)] of *uncoupled* oscillators at different M ranging from 2 to 10. Which concrete oscillators are taken as members of an ensemble of size $M<10$ is not important since the results appear very similar for any choice. In particular, in Fig. 7 we report the results for the following "centered" ensembles: oscillators 5 and 6 for $M=2$, oscillators 4–6 for $M=3$, oscillators 4–7 for M

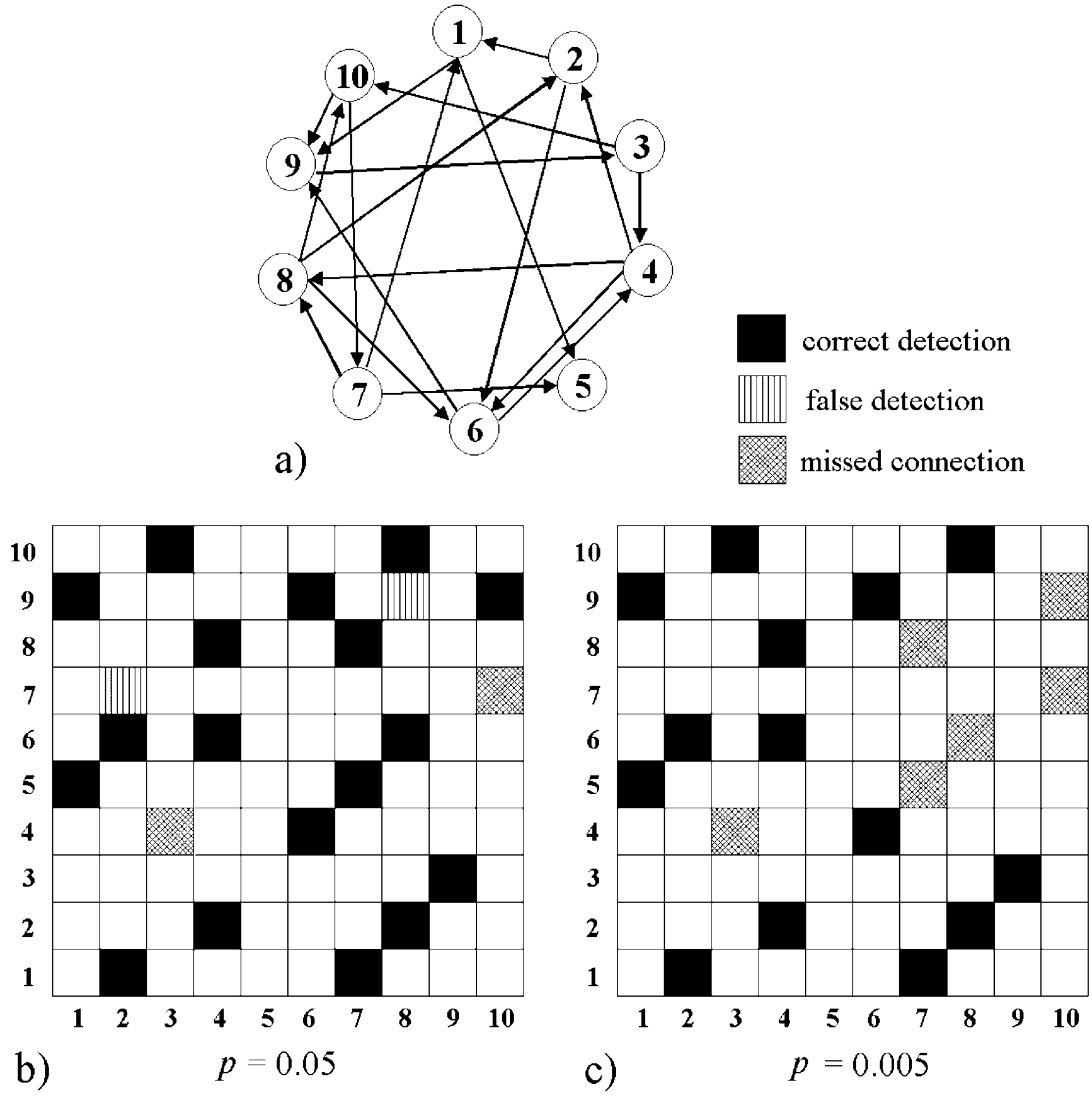

FIG. 6. Coupling estimation for an ensemble of 10 phase oscillators [Eq. (2)] with "random" couplings from a single time series. Notations are the same as in Fig. 4: (a) coupling architecture; [(b) and (c)] diagrams for coupling estimation results.

=4, and so on. Since oscillators are uncoupled, all couplings detected are false positives. A total rate of false positives ν_{err} at $p=0.05$ is calculated at various time series lengths N for each M and a minimal length N_{min} providing $\nu_{err}<0.05$ within acceptable limits [55] is determined. As expected, a necessary amount of data rises with an ensemble size (Fig. 7). To mention a few typical lengths, 40 basic periods suffice for $M=2$ or $M=3$ while one needs at least 300 basic periods for $7 \leq M \leq 10$.

C. Other stochastic and chaotic oscillators

We test the technique with ensembles of stochastically perturbed limit-cycle oscillators (van der Pol oscillators), deterministically chaotic oscillators (Roessler systems), and a model of an active spatially distributed nonuniform nonlinear medium (Ginzburg-Landau equation). In all cases, the phases are computed via Hilbert transform [50] and ten basic periods at each edge are then ignored to reduce edge effects [51]. As shown below, the suggested estimators appear applicable and sensitive.

An ensemble of van der Pol oscillators with ring coupling architecture [Fig. 4(a)] is given by the equations

$$\ddot{x}_k = (0.2 - x_k^2)\dot{x}_k - \omega_k^2 x_k + K_{j\to k}(x_j - x_k) + \xi_k, \qquad (9)$$

where $k=1,\ldots,10$, $j=k-1$ for $k>1$, $j=10$ for $k=1$, $\omega_k = 1.275-0.05k$, and $\sigma_{\xi_k}^2=0.04$. Individual dynamics of each oscillator for $\sigma_{\xi_k}^2=0$ are represented by periodic self-sustained oscillations so that the phase dynamics is accurately described by the phase oscillator [Eq. (2)]. The presence of noises ξ_k leads to considerable fluctuations of amplitudes. Therefore, the phase dynamics of noisy oscillators are only approximately described with the phase oscillator [Eq. (2)]. Still, the approximation is reasonably good so that the suggested estimators appear quite efficient.

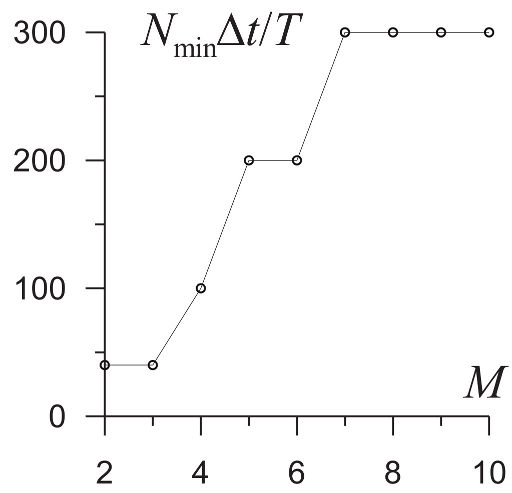

FIG. 7. Coupling estimation for ensembles of phase oscillators of different size. A minimal time series length providing controlled rate of false positives is shown versus an ensemble size.

Analysis of 100 time series from an ensemble of uncoupled oscillators ($K_{j\to k}=0$) gives the rate of false positives at $p=0.05$ equal to 0.054 when the value 0.055 is acceptable [55]. If couplings exist, they are detected correctly stronger couplings being detected more readily. For instance, in case of weak uniform coupling strength $K_{j\to k}=0.03$ (weakness of such couplings is reflected in that $\hat{\rho}_{j,k}<0.25$ for all j,k) probability to detect each existing connection is equal to 0.55, a typical example is shown in Fig. 8(a).

Equations for an ensemble of Roessler systems with ring coupling [Fig. 4(a)] read

$$\dot{x}_k = -\omega_k y_k - z_k + K_{j\to k}(x_j - x_k),$$

$$\dot{y}_k = \omega_k x_k + 0.398 y_k,$$

$$\dot{z}_k = 2.0 + (x_k - 4.0) z_k, \qquad (10)$$

where $k=1,\ldots,10$, $j=k-1$ for $k>1$, $j=10$ for $k=1$, and $\omega_k=1.275-0.05k$. Individually, such a system demonstrates spiral chaos for a frequency value of $\omega_k=1.0$ [57]. Frequency variations may change the properties of chaotic behavior by shifting a system to another domain in a parameter space. In any case, the phase dynamics can be described with Eq. (2) where irregularity is determined by a chaotic amplitude fluctuations whose statistics differs from white noise properties.

As we checked for other parameter values and/or incorporation of Gaussian noise into Eq. (10), ACFs of the residuals ε_k decay fast enough [e.g., down to the level of 0.1–0.2 over an interval $(0,\tau)$]. Thus, applicability of our estimators is guaranteed and, indeed, they perform properly in such cases (not shown since the results are quite analogous to the previous examples). However, in the case of Eq. (10) distribution of the residuals ε_k is non-Gaussian and, even more important, ACF decay time exceeds τ. Thus, applicability of the suggested estimators is not assured and it is interesting to check whether they are inevitably erroneous.

Analysis of 100 time series from an ensemble of uncoupled oscillators ($K_{j\to k}=0$) exhibits the rate of false positives at $p=0.05$ equal to 0.04 when the value 0.055 is acceptable [55]. If couplings exist, they are again detected correctly. For instance, in case of weak uniform coupling strength $K_{j\to k}=0.01$ (all $\hat{\rho}_{j,k}$ are less than 0.1) probability to detect each existing connection is equal to 0.7, a typical example is shown in Fig. 8(b). Thus, the suggested estimators are again applicable that is an evidence for their robustness, i.e., formulas derived under the assumption of white noises ξ_k apply under some variations in the "noise" statistical properties (see, e.g., [58] for possible theoretical explanations).

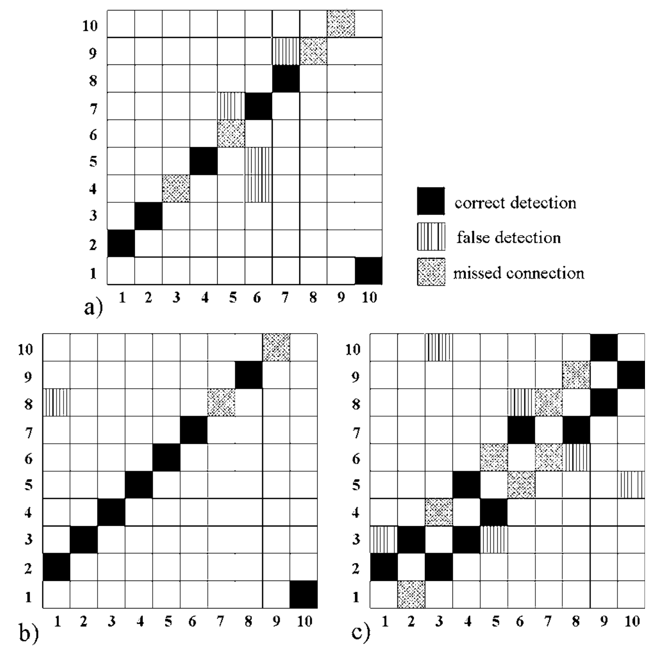

FIG. 8. Coupling estimation for ensembles of different oscillators from a single time series of the length of 300 basic periods at $p=0.05$. Notations are the same as in Fig. 4: (a) van der Pol oscillators with ring couplings (9); (b) Roessler systems with ring couplings [Eq. (10)]; and (c) ten points of a medium [Eq. (11)].

Yet, it should be noted that the technique is assured to work properly only if the conditions listed in Sec. III are fulfilled. Otherwise, as in case (10), the technique may be used as a heuristic tool so that its results should be taken with caution. Thus, at other parameter values where the applicability condition with respect of ε_k properties is also violated, we have observed the rate of false positives significantly exceeding the required value of p.

Finally, we consider a more complicated example representing a model of active nonlinear nonuniform continuous medium described by a partial differential equation. This is widespread in different fields of physics, Ginzburg and Landau equations (see, e.g., [59])

$$\frac{\partial a(x,t)}{\partial t} = i\omega(x)a + \frac{(1-|a|^2)a}{2} + g\frac{\partial^2 a}{\partial x^2} + \xi(x,t), \qquad (11)$$

where a spatial coordinate $0\le x\le 10$, a is a complex amplitude, a frequency depends on a spatial coordinate $\omega(x)=0.8+0.04x$, a diffusion coefficient g determines coupling between "neighboring points" of a medium, boundary conditions are $\partial a/\partial x=0|_{x=0,10}$, and $\xi(x,t)$ are (spatially) independent sources of white noise with intensity σ_ξ^2.

A stochastic partial differential equation is a quite complicated object both for numerical solution and theoretical consideration. Here, we have used a rough approach including spatial discretization with a step 0.1, replacement of spatial derivatives by finite differences $\partial^2 a(x,t)/\partial x^2 \approx [a(x-0.1,t)-2a(x,t)+a(x+0.1,t)]/0.01$, and difference Euler scheme for temporal integration, i.e., we have solved 100 coupled ordinary first-order differential equations for complex variables $a(x,t)$. At the specified parameter values, the variables $\mathrm{Re}[a(x,t)]$ exhibit self-sustained oscillations at each point x. "Observed" time series are $\mathrm{Re}[a(x,t)]$ at points $x_k=k-0.5$, $k=1,\ldots,10$. Among those processes, any two "neighboring" ones are bidirectionally coupled to each other while the couplings are mediated by multiple intermediate points of the medium whose oscillations are not observed. The conditions for applicability of the suggested estimators seem to be fulfilled.

The rough numerical integration technique does not pretend to give a very strict and accurate description of the dynamics of system (11). However, even being somewhat different from Eq. (11), such a numerical scheme also represents a model of a continuous medium. Hence, for our purposes it is sufficient to be used as a more complicated test allowing to check an effect of "hidden" intermediate units in an ensemble.

The suggested estimators perform well at different values of the diffusion coefficient g providing reasonable sensitivity and the rate of false positives within acceptable limits. Thus, at $g=0.003$ and $\sigma_\xi^2=0.05$ (coupling is weak so that all $\hat{\rho}_{j,k} < 0.3$) the rate of correct coupling detections is 0.5 which is quite a considerable value. A typical example is shown in Fig. 8(c).

V. CONCLUSIONS

We have suggested estimators of couplings in ensemble of oscillators based on phase dynamics modeling. They allow us to reveal architecture of couplings between *noisy* oscillators from a single time series. An analytic significance level is derived to detect couplings reliably that is important for applications.

The main conditions for the approach applicability can be summarized as follows:

(i) phases of the oscillators are well defined and phase oscillator model (2) is adequate;

(ii) couplings are not strong enough to induce considerable synchrony between any two oscillators; and

(iii) a time series length is not less than several dozens of basic periods and depends on an ensemble size as shown in Fig. 7.

The results are validated with different numerical examples including a model of an active nonlinear medium. Thus, the suggested approach makes a fruitful idea of phase dynamics modeling essentially wider applicable in practice for the analysis of couplings in oscillatory ensembles.

ACKNOWLEDGMENTS

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