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Estimating the Parameters of Semiconductor Optical-Feedback Lasers from Time Series

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Abstract—A new method is proposed for estimating the control parameters of single-mode semiconductor optical-feedback lasers from experimentally measured time series of the laser intensity. The procedure is based on a statistical analysis of specially selected points of the time series and on the phenomenon of chaotic synchronization of unidirectionally coupled lasers with coinciding parameters. *© 2005 Pleiades Publishing, Inc.*

In recent years, much attention has been devoted to the use of semiconductor optical-feedback lasers for secure data transmission [1–3]. The interest in these lasers as potential sources for data transmission systems with a high degree of information security is related to the fact that the laser intensity exhibits wideband chaotic oscillations of very high dimension. The extraction of transmitted data in communication systems employing chaotic signals is possible due to the phenomenon of synchronization of coupled chaotic dynamical systems [4–6]. Such a synchronization of the interacting chaotic systems is manifested for certain types of coupling between them and can be used to estimate the control parameters of the coupled systems [7–12]. Previously, several methods were proposed for the estimation of parameters of coupled dynamical systems, which were based on autosynchronization [7], adaptive control [8, 12], random optimization [9], error minimization [10], and iterative adaptation of parameters [11].

This Letter describes a new method for sequential refinement of the control parameters of semiconductor optical-feedback lasers, which is based on the phenomenon of chaotic synchronization.

Let us consider a single-mode semiconductor optical-feedback laser whose operation is described by the Lang–Kobayashi equations [13]. After appropriate normalization, these equations can be written in the following form [14]:

$$\begin{aligned} \dot{E}(t) &= (1+i\alpha)F(t)E(t) \\ &\quad + \eta E(t-\tau_0)\exp(-i\Omega\tau_0), \\ T\dot{F}(t) &= P - F(t) - (1+2F(t))|E(t)|^2, \end{aligned} \qquad (1)$$

where $E(t)$ is the complex amplitude of the electric field, which exhibits slow variation (on the time scale of optical oscillations); F is the density of nonequilibrium charge carriers; $T = \tau_s/\tau_p$ is the ratio of the carrier lifetime τ_s to the photon lifetime τ_p in the laser cavity; P is the pumping parameter (in excess of the generation threshold); τ_0 is the delay time in the optical feedback chain; η is the feedback gain; α is the factor of nonisochronicity; Ω is the laser frequency in the absence of the feedback; and the upper dot denotes differentiation with respect to dimensionless time t (expressed in units of the photon lifetime τ_p).

Writing the complex field amplitude as $E(t) = \rho(t)\exp(i\phi(t))$ [where $\rho(t)$ and $\phi(t)$ are the modulus and phase of $E(t)$], denoting $\rho = \rho(t)$, $\rho_\tau = \rho(t-\tau_0)$, $\phi = \phi(t)$, $\phi_\tau = \phi(t-\tau_0)$, and $F = F(t)$, and taking into account that the complex constant $\exp(-i\Omega\tau_0)$ determining the phase shift in the feedback chain does not qualitatively influence the procedure proposed below for evaluation of the delay time (and, hence, can be omitted), we can rewrite system of equations (1) as follows:

$$\begin{aligned} \dot{\rho} &= F\rho + \eta\rho_\tau\cos(\phi-\phi_\tau), \\ \rho\dot{\phi} &= F\alpha\rho - \eta\rho_\tau\sin(\phi-\phi_\tau), \\ T\dot{F} &= P - F - (1+2F)\rho^2. \end{aligned} \qquad (2)$$

Previously, we demonstrated that the time series of delay systems of the type $\dot{x}(t) = F(x(t), x(t-\tau))$ possess virtually no extrema separated from each other by τ [15] and developed a method employing this circumstance for determining τ. Now we will show that this method, after appropriate modification, can also be applied to a time-delay system of the type described by Eqs. (2). Differentiating the first equation in system (2) with respect to time t, we obtain

$$\begin{aligned} \ddot{\rho} &= \dot{F}\rho + F\dot{\rho} + \eta\dot{\rho}_\tau\cos(\phi-\phi_\tau) \\ &\quad - \eta\rho_\tau\sin(\phi-\phi_\tau)(\dot{\phi}-\dot{\phi}_\tau). \end{aligned} \qquad (3)$$

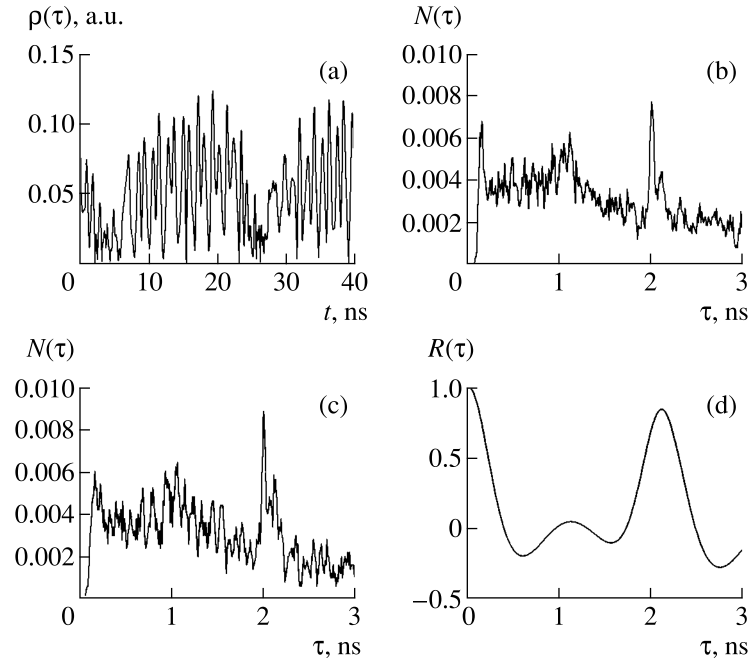

Fig. 1. Estimation of the parameters of a Lang–Kobayashi system (2): (a) typical experimental time series for the system with parameters $P = 1.1 \times 10^{-3}$, $\tau_0 = 2$ ns, $\eta = 5 \times 10^{-3}$, $T = 10^3$, and $\alpha = 5$; (b, c) plots of $N(\tau)$ for the time series of variables ρ and I normalized to the total number of points simultaneously obeying conditions (4) and (6); (d) autocorrelation function $R(\tau)$ for the laser intensity oscillations.

The typical time series of oscillations of the variable $\rho = |E|$ for a single-mode semiconductor optical-feedback laser described by Eqs. (2) with a small value of parameter P exhibits a large number of extrema in which ρ is close to zero (Fig. 1a). Such points are characterized by simultaneously obeying two conditions:

$$\dot{\rho} = 0, \quad \rho < \varepsilon, \tag{4}$$

where ε is a certain small quantity. According to Eq. (3), these points also obey the relation

$$\frac{\dot{\rho}_\tau}{\rho_\tau} \approx \frac{\ddot{\rho} + \eta\rho_\tau \sin(\phi - \phi_\tau)(\dot{\phi} - \dot{\phi}_\tau)}{\eta\rho_\tau \cos(\phi - \phi_\tau)}. \tag{5}$$

In the typical case of quadratic extrema, we have $\dot{\rho} = 0$ and $\ddot{\rho} \neq 0$. From this we infer that, for small ρ_τ, the ratio $\dot{\rho}_\tau/\rho_\tau$ must be quite large. Taking this fact into account, we can use the following method of processing of the time series of ρ to determine the delay time τ_0 in a system described by equations (2). First, we consider the points satisfying conditions (4) and select those also meeting the conditions

$$\rho_\tau < \varepsilon, \quad \dot{\rho}_\tau/\rho_\tau > \Theta, \tag{6}$$

for sufficiently large Θ. Since τ_0 is unknown, we try various τ and count the number N of the pairs of points simultaneously satisfying conditions (4) and (6). Then, the function $N(\tau)$ represents the number of the pairs of points for which conditions (4) are satisfied at the time t and conditions (6) at $t - \tau$. For τ equal to the true delay time τ_0, the number of points $N(\tau_0)$ must be greater than that for arbitrary τ, where the conditions (6) may not be satisfied. Therefore, the position of the maximum in the $N(\tau)$ curve gives us an estimate for the delay time τ_0. The same considerations are applicable to the experimental time series of oscillations of the laser intensity $I = |E|^2$.

Figure 1b shows the $N(\tau)$ curve constructed as described above for a time series of the variable ρ in Eqs. (2) with $\tau_0 = 2$ ns, $P = 1.1 \times 10^{-3}$, $\eta = 5 \times 10^{-3}$, $T = 10^3$, $\alpha = 5$, $\varepsilon = \sigma_1/k_1$, and $\Theta = \sigma_2 k_2$, where σ_1 is the rms deviation of ρ, σ_2 is the rms deviation of $\dot{\rho}_\tau/\rho_\tau$, $k_1 = 1$, and $k_2 = 7$. This plot was constructed using a series of 400000 points, which contained about 4000 points satisfying conditions (4). As the values of k_1 and k_2 are increased, the number of points used for the construction of the $N(\tau)$ curve decreases. When τ was varied at a step of 0.01 ns, the absolute maximum of $N(\tau)$ is observed at $\tau = 2.00$ ns, which is precisely the actual delay time. Figure 1c presents a plot of $N(\tau)$ constructed using time series of the intensity I for system (2) with the same control parameters as in Fig. 1b, but with $k_1 = k_2 = 4$. In this case, the procedure of delay time estimation based on the search for a maximum of the autocorrelation function yields $\tau_0 = 2.10$ ns (Fig. 1d).

For estimating the other parameters of the system under consideration, we suggest the phenomenon of chaotic synchronization observed for two semiconductor lasers with unidirectional coupling be used. Such unidirectionally coupled systems (1) are described by the following equations [16]:

$$\begin{aligned} \dot{E}^{T,R} &= (1 + i\alpha^{T,R})F^{T,R}E^{T,R} \\ &+ \eta^{T,R}E^{T,T}(t-\tau^{T,R})\exp(-i\Omega^{T,R}\tau^{T,R}), \qquad (7) \\ T^{T,R}\dot{F}^{T,R} &= P^{T,R} - F^{T,R} - (1+2F^{T,R})|E^{T,R}|^2, \end{aligned}$$

where the variables and parameters of the drive system (transmitter) are indicated by superscript "T" and those of the response system (receiver) are indicated by superscript "R." When the parameters of two coupled systems are sufficiently close, these systems exhibit synchronization [16].

Since the α and T values in real lasers of the same type are fixed and close, we assume these parameters for the transmitter and receiver to be same: $\alpha^T = \alpha^R$ and $T^T = T^R$. The other parameters of the transmitter (P^T, η^T, and τ^T) will be estimated by providing its synchronization with the receiver, whose parameters (P^R, η^R, and τ^R) can be tuned. A convenient quantitative measure of synchronization is provided by the correlation function

$$C(\tau) = \frac{\langle I^T(t)I^R(t+\tau)\rangle}{\sqrt{\langle I^T(t)^2\rangle\langle I^R(t)^2\rangle}}, \qquad (8)$$

where $I^{T,R} = |E^{T,R}|^2$ and the angle brackets indicate averaging with respect to time. In the case of compete synchronization, this function has a maximum value at the origin, and this maximum reaches unity when all parameters of the transmitter and receiver are identical. If the analogous parameters of the transmitter and receiver are slightly different, the maximum value of $C(\tau)$ differs from unity, while its shift from the origin gives an estimate of the difference of delay times in the transmitter and receiver.

Let us set the same transmitter parameters as those used above for a single system and select the initial approximation as $\tau^R = 2.00$ ns (which corresponds to the absolute maximum of $N(\tau)$) and $\eta^R = 1 \times 10^{-3}$. The third parameter ($P^R$) will be estimated by plotting the maximum value (C_{max}) of the correlation function (8) versus the parameter P^R (Fig. 2a). As can be seen, C_{max} reaches the absolute maximum at $P^R = 1.8 \times 10^{-3}$. Then, we plot C_{max} as a function of η^R for $P^R = 1.8 \times 10^{-3}$ (Fig. 2b). This curve has the absolute maximum at $\eta^R = 5.2 \times 10^{-3}$. In the next step, we again plot C_{max} versus P^R, but with the just refined value of η^R (Fig. 2c), and so on. The next step (Fig. 2d) already gives the true value of the feedback gain, $\eta^R = \eta^T = 5.0 \times 10^{-3}$, for which the P^R value no longer varies in subsequent iteration steps. The correlation function exhibits maximum at the origin and this maximum is $C_{max} = 1$. This result confirms that the plot of $N(\tau)$ constructed as described above provides exact estimation of the delay time, since $\tau^R = \tau^T = 2$ ns.

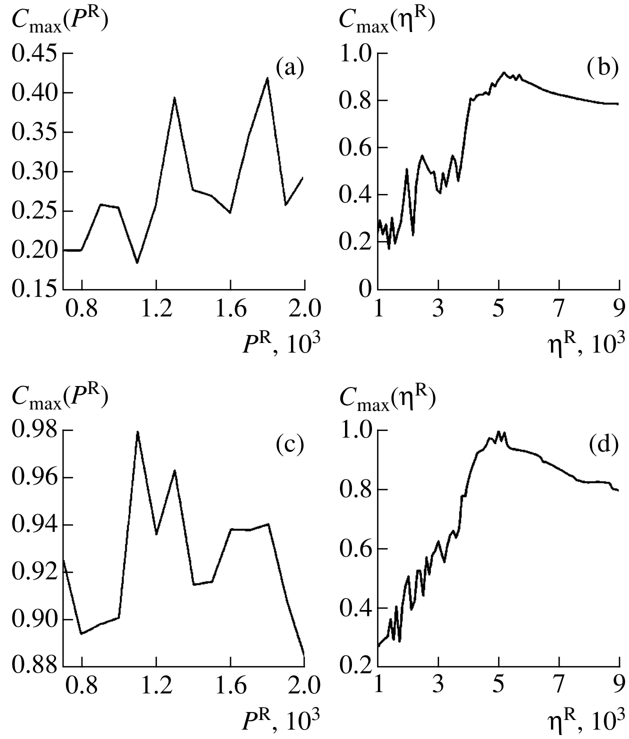

Fig. 2. Sequential refinement of the transmitter parameters based on the chaotic synchronization of unidirectionally coupled lasers as illustrated by plots of the maximum (C_{max}) of the correlation function (8) versus the receiver parameters P^R and η^R: (a) $\eta^R = 1 \times 10^{-3}$, $C_{max}(P^R)$ is maximum at $P^R = 1.8 \times 10^{-3}$; (b) $P^R = 1.8 \times 10^{-3}$, $C_{max}(\eta^R)$ is maximum at $\eta^R = 5.2 \times 10^{-3}$; (c) $\eta^R = 5.2 \times 10^{-3}$, $C_{max}(P^R)$ is maximum at $P^R = 1.1 \times 10^{-3}$; (d) $P^R = 1.1 \times 10^{-3}$, $C_{max}(\eta^R)$ is maximum at $\eta^R = 5.0 \times 10^{-3}$.

Applying the proposed method for the numerical investigation of system (7), it is also possible to estimate the other parameters of the transmitter, for example, α^T and T^T, under the conditions $\eta^R = \eta^T$ and $P^R = P^T$. If all parameters of the transmitter in the numerical experiment are unknown, the procedure converges to the true values only provided that the initial values are selected close to the corresponding true parameters.

We have also evaluated the robustness of the above procedure of sequential refinement of the control parameters by introducing an additive noise into the communication channel between the transmitter and receiver. It was established that the proposed method provides correct estimation of the parameters P^T and η^T even in the presence of significant noise on a level of 10%.

Conclusion. We have developed a new method for the sequential refinement of parameters of single-mode semiconductor optical-feedback lasers described by the Lang–Kobayashi equations. The proposed procedure employs the phenomenon of chaotic synchronization of unidirectionally coupled lasers. We also described a procedure for the initial estimation of the delay time in the feedback chain, which is based on the statistical analysis of specially selected points of a measured time series of the laser intensity. The efficiency of the proposed method is confirmed by the results obtained for unidirectionally coupled Lang–Kobayashi systems.

The possibility of estimating the parameters of semiconductor optical-feedback lasers provides a means of useful signal extraction in communication systems employing chaotic signals for masking the transmitted data. Therefore, the level of security provided by such communication systems based on single-mode semiconductor lasers can be insufficient, despite the very high dimension and the large number of positive Lyapunov exponents for chaotic attractors in these dynamical systems.

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