

Characterizing direction of coupling from experimental observations

Boris Bezruchko and Vladimir Ponomarenko

Institute of Radio-Engineering and Electronics, Russian Academy of Sciences, Zelyonaja 38, 410019 Saratov, Russia

Michael G. Rosenblum and Arkady S. Pikovsky

Department of Physics, University of Potsdam, Am Neuen Palais, PF 601553, D-14415, Potsdam, Germany

(Received 8 May 2002; accepted 9 September 2002; published 21 February 2003)

We demonstrate that the direction of coupling of two interacting self-sustained electronic oscillators can be determined from the realizations of their signals. In our experiments, two electronic generators, operating in a periodic or a chaotic state, were subject to symmetrical or unidirectional coupling. In data processing, first the phases have been extracted from the observed signals and then the directionality of coupling was quantitatively estimated from the analysis of mutual dependence of the phase dynamics. © 2003 American Institute of Physics. [DOI: 10.1063/1.1518425]

Traditional methods of bivariate data analysis like computation of cross-spectra, cross-correlation, or mutual information provide symmetric measures of interrelation between the signals and do not allow one to address the question of causality in the interaction. Recently, several approaches to this problem have been suggested, based either on the notion of generalized synchronization¹⁻⁵ or on the information theory and Granger's concept of causality.⁶⁻⁸ Here we discuss a method that is based on the coupled oscillators theory; it is aimed to quantify the degree of asymmetry in interaction. This method is designed for the data generated by two weakly coupled self-sustained oscillators. This limitation is compensated by the simplicity of the method and the absence of parameters. In the paper the method is applied to experimental data from coupled periodic and chaotic electronic circuits.

I. INTRODUCTION

Coupled oscillators are ubiquitous in science, nature, engineering, and social life. Examples include pendulum clocks, firing neurons, lasers, singing crickets, etc.⁹⁻¹³ Quite often the degree of coupling can be directly controlled in an experiment, allowing one to characterize coupling-dependent effects, such as synchronization, oscillation death, beats, etc. The inverse problem, i.e., the determination of the level and the direction of coupling from the observations of coupled systems, is much less trivial. For example, from the observation of a synchronous motion one can hardly detect the direction of coupling, because both symmetrical (bidirectional) and unidirectional coupling can yield synchronization of oscillators.

In this paper we present an experimental realization of a recently suggested method^{14,15} for determining the direction of coupling from the signals coming from interacting oscillating systems. In our setup we use two electronic generators, operating in a periodic or a chaotic state. Moreover, in the experiment it is possible to perturb the systems by external noise and in this way to test the applicability of the method

under various conditions. The paper is organized as follows. In Sec. II we describe the method of the analysis of bivariate data. In Sec. III the experimental setup is described. The main results of the paper—detection of the direction of coupling from the experimentally observed data—are presented in Sec. IV. We conclude with a discussion of the limitations and the drawbacks of the method.

II. DESCRIPTION OF THE METHOD

The main idea of Ref. 14 is to use a general property that a weak coupling affects the phases of interacting oscillators, whereas the amplitudes remain practically unchanged.^{11,16} Hence, the dynamics of weakly coupled oscillators can be reduced to those of two phases $\phi_{1,2}$:

$$\begin{aligned}\dot{\phi}_1 &= \omega_1 + \epsilon_1 f_1(\phi_1, \phi_2) + \xi_1(t), \\ \dot{\phi}_2 &= \omega_2 + \epsilon_2 f_2(\phi_2, \phi_1) + \xi_2(t).\end{aligned}\tag{1}$$

Here, random terms $\xi_{1,2}$ describe noisy perturbations that are always present in real-world systems, small parameters $\epsilon_{1,2} \ll \omega_{1,2}$ characterize the strength of the coupling. Equation (1) also describes the phase dynamics of coupled continuous-time chaotic systems; in this case $\xi_{1,2}$ are irregular terms that reflect the chaotic nature of amplitudes.¹⁷ The fact that the regular component of the phase dynamics is two-dimensional, essentially simplifies detection of the asymmetry in the interaction.

Functions $f_{1,2}$ are 2π -periodic in both arguments and combine the description of the phase dynamics of autonomous (uncoupled) systems and the coupling between them. If the coupling is bidirectional, f_1 and f_2 depend on both ϕ_1 and ϕ_2 . In the case of a unidirectional driving, say from system 1 to system 2, $f_1 = f_1(\phi_1)$, whereas $f_2 = f_2(\phi_1, \phi_2)$ is the function of two arguments.

In order to estimate the dependencies of the functions $f_{1,2}$ it is convenient to go from a continuous-time description of the process to a time-discrete one. Two methods have been proposed in the literature,^{14,15} hereafter we use the so-called “instantaneous period approach” described in Ref. 15. In this method one first obtains the phases as functions of con-

tinuous time: $\phi_1(t)$, $\phi_2(t)$. For good oscillators the phase grows monotonically in time, thus for each instant t_k one can define two time intervals $T_1(k)$ and $T_2(k)$ at which the phases ϕ_1 and ϕ_2 gain 2π , respectively:

$$\begin{aligned}\phi_1(t_k + T_1(k)) &= \phi_1(t_k) + 2\pi, \\ \phi_2(t_k + T_2(k)) &= \phi_2(t_k) + 2\pi.\end{aligned}\quad (2)$$

These time intervals depend, in general, on both phases; also the noisy part (terms $\eta_{1,2}$) may be present:

$$\begin{aligned}T_1(k) &= \Theta_1(\phi_1(k), \phi_2(k)) + \eta_1, \\ T_2(k) &= \Theta_2(\phi_2(k), \phi_1(k)) + \eta_2.\end{aligned}\quad (3)$$

If the functions $\Theta_{1,2}$ are known (or estimated from the observed data), we can characterize the dependence of the local “period” $T_{1,2}$ on the other phase via the mean square of the partial derivative:

$$c_1^2 = \left\langle \left(\frac{\partial \Theta_1}{\partial \phi_2} \right)^2 \right\rangle, \quad c_2^2 = \left\langle \left(\frac{\partial \Theta_2}{\partial \phi_1} \right)^2 \right\rangle.\quad (4)$$

Finally, the *directionality index* is introduced as

$$r = \frac{c_2 - c_1}{c_2 + c_1}.\quad (5)$$

In the case of unidirectional coupling $r = \pm 1$, while for nearly symmetrical coupling r is close to zero. Note that the notion “symmetrical” is not well-defined: terms of the same order in the original equations of motion may lead to different contributions to the phase dynamics, due to difference in the periods, amplitudes, forms of oscillations, etc. Nevertheless, small values of r indicate that the coupling is effectively bidirectional.

We note that the method is explicitly based on the assumption that one deals with weakly interacting self-sustained oscillators. Recently, other methods for evaluation of causality in interrelation of bivariate data have been suggested, where no assumptions on the systems under investigation are required. One approach, based on the information theory, uses entropy measures.^{7,8} A second approach, arising from studies of generalized synchronization, exploits the idea of mutual predictability: it quantifies the ability to predict the state of the first system from the knowledge of the second one.¹⁻⁵

According to the above-described method, the algorithm for detecting the directionality of coupling can be outlined as follows.

(1) One obtains the phases $\phi_{1,2}(t)$ of the signals from the two oscillators. This task may be nontrivial, although the corresponding methods (as well as possible difficulties) are well documented in the literature (see, e.g., Ref. 11, and references therein). Usually, a preprocessing is needed depending on the structure of the signals under consideration.

(2) For a set of time instants t_k one obtains the instantaneous “periods” according to Eq. (2). These periods are considered as unknown functions of the phases at time t_k .

(3) One estimates the functions $\Theta_{1,2}(\phi_{1,2}, \phi_{2,1})$ (3), using some fit with time series $T_{1,2}$ and $\phi_{1,2}$. Because the functions $\Theta_{1,2}$ are 2π -periodic, a Fourier fit is suitable.

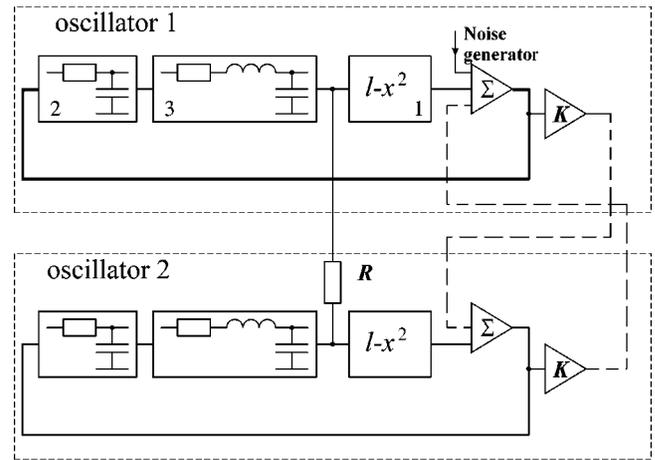


FIG. 1. Scheme of the experimental setup. Each generator consists of non-linear element (1), first-order filter (2), second-order filter (3), and summing amplifier (Σ). The feedback (shown by the bold line) is implemented via connection of the output of the summing Σ to the input of the filter (2). Symmetric coupling is provided by the resistor R ; unidirectional coupling (dashed lines) is implemented via additional amplifiers K .

(4) From the approximated functions $\Theta_{1,2}$ one calculates the cross-dependencies $c_{1,2}$ according to Eq. (4). Finally, the directionality index (5) is obtained.

III. EXPERIMENTAL SETUP

In the experiment we use two electronic self-sustained chaotic oscillators, which are coupled either bidirectionally or unidirectionally. Each oscillator is constructed as a ring consisting of a low pass RC filter, resonant RLC filter, non-linear element, and a summing-amplifier (Fig. 1). The non-linear element is the multiplier, whose output signal v_{out} is related to the input signal v_{in} as $v_{\text{out}} = l - mv_{\text{in}}^2$. Here $m = 0.1 \text{ V}^{-1}$ and l is the external voltage governing the non-linearity level in the circuit and serving as the main bifurcation parameter. Depending on this parameter, the oscillator can demonstrate periodic or chaotic oscillations; the transition to chaos happens via a period-doubling cascade (Fig. 2). Additionally, there is a possibility to add noise to the circuit; this is implemented by adding a signal from the noise generator to the summing-amplifier.

Coupling of two generators is accomplished in two ways. To implement a nearly symmetrical coupling, the points at the input of the nonlinear elements in two circuits are connected via a resistor (denoted as R in Fig. 1). The coupling is thus inversely proportional to the resistance. To

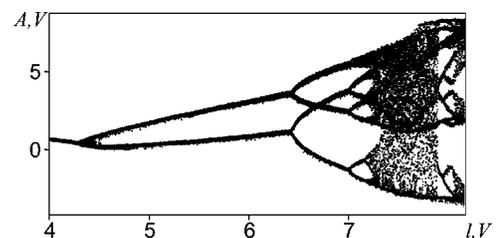


FIG. 2. Bifurcation diagram of the first oscillator. A is the voltage on the output of the amplifier, l is the parameter of nonlinearity.

TABLE I. Results of the estimation of the quantities (4,5) from experimental data. “Bidirectional” coupling means coupling via the resistor R (values are given in $k\Omega$); “unidirectional” means coupling via the amplifier (positive K means that it is directed from circuit 1 to circuit 2); “bidirectional and unidirectional” means a combination of these couplings. Parameters of nonlinearity of both generators $l_{1,2}$ are given in volts. The details of preprocessing are described in the text.

Coupling	Preprocessing	c_1	c_2	r
Cycle–cycle				
Bidirectional ($l_1=3, l_2=1.4, R=6$)		0.12	0.27	0.39
Unidirectional ($l_1=3, l_2=1.4, K=0.1$)		0.01	0.35	0.95
Unidirectional, with noise	Filter 51 points	0.05	0.52	0.82
Unidirectional, with noise	Filter 25 points	0.25	0.58	0.40
Double revolution cycles				
Unidirectional ($l_1=6, l_2=4, K=0.1$)		0.11	0.35	0.51
Unidirectional	Running average removed	0.02	0.06	0.74
Bidirectional ($l_1=5.8, l_2=4, R=16$)		0.98	1.10	0.06
Bidirectional	Running average removed	0.16	0.18	0.08
Bi- and unidirectional ($l_1=6, l_2=4.8, R=15, K=0.1$)		0.40	0.50	0.12
Bi- and unidirectional	Running average removed	0.06	0.07	0.08
Chaos–chaos				
Bidirectional ($l_1=7.6, l_2=6.4, R=33$)	Running average removed	0.13	0.15	0.06
Bidirectional (almost synchrony) ($l_1=7.6, l_2=6.4, R=32$)	Running average removed	22.71	18.58	-0.1
Bi- and unidirectional (almost synchrony) ($l_1=7.6, l_2=6.4, R=33, K=0.1$)	Running average removed	2.22	3.44	0.22
Unidirectional from 2 to 1 ($l_1=8.5, l_2=6.2, K=-0.2$)	Bandpass filter	0.08	0.01	-0.78

implement a unidirectional coupling, a signal from the output of the summator in one circuit is added to the input of the summator in the other circuit via the amplifier K ; these connections are marked by dashed lines in Fig. 1. The level of this unidirectional coupling can be regulated by the additional amplifier K . Combination of two couplings provides asymmetric interaction between two generators.

We have not made special efforts to make the generators identical, thus their mean frequencies (around 14 kHz) differ by $\approx 10\%$. Time series were recorded into computer memory using an analog-to-digital converter. Sampling frequency 500 kHz is 20–30 times greater than the characteristic frequency of the oscillator. The length of each signal was 100 000 points; on the following plots the signals are shown in arbitrary units.

IV. CHARACTERIZING DIRECTION OF COUPLING

In this section we present the results of application of the method described in Sec. II to the data obtained in experiments with coupled electronic generators (see Sec. III). As each oscillator can operate in different dynamical regimes, we can explore the performance of the analysis technique in various situations (e.g., cycle–cycle, cycle–chaos, etc.). In the following we describe the characteristic cases.

The first step in the analysis is the estimation of instantaneous phase $\phi(t)$ from each time series. For this goal we exploit the Hilbert transform (for description and technical details see Ref. 11) and compute the phase as

$$\phi = \arctan\left(\frac{x_h}{x - x^*}\right), \quad (6)$$

where x is the original signal, x_h is its Hilbert transform (HT), and the constant x^* is chosen via visual inspection of the x_h vs x plots, to ensure rotation of all trajectories around the origin on the plane $(x - x^*, x_h)$. In almost all cases, the data are filtered prior to the application of the HT. This is a crucial point of data analysis that strongly influences the performance of the phase estimation. The choice of a proper filter depends on the data, we illustrate this by description of particular examples.

A. Cycle–cycle

We start with a case when both oscillators, being uncoupled and without noisy perturbations, exhibit limit cycle oscillations. The results are summarized in Table I. In the noise-free case we obtain $r=0.39$ for the case of the bidirectional and $r=0.95$ for the unidirectional coupling. For the bidirectional coupling the value of the directionality index is essentially different from zero due to two reasons: different frequencies and wave forms of signals. We recall here that for nonidentical systems the definition of symmetric coupling is ambiguous.

Next, we consider systems perturbed by noise (in this experiment noise was added to the first circuit). As the noise-free generators are close to the period doubling point, perturbation results in a rather complex form of the signals, see Fig. 3. Here noise reduction is needed, thus the data have been smoothed by the Savitzky–Golay (polynomial) filter of fourth order. We have found that the directionality index depends strongly on the filter characteristics. In particular, for unidirectional coupling and the filter length 51 points the

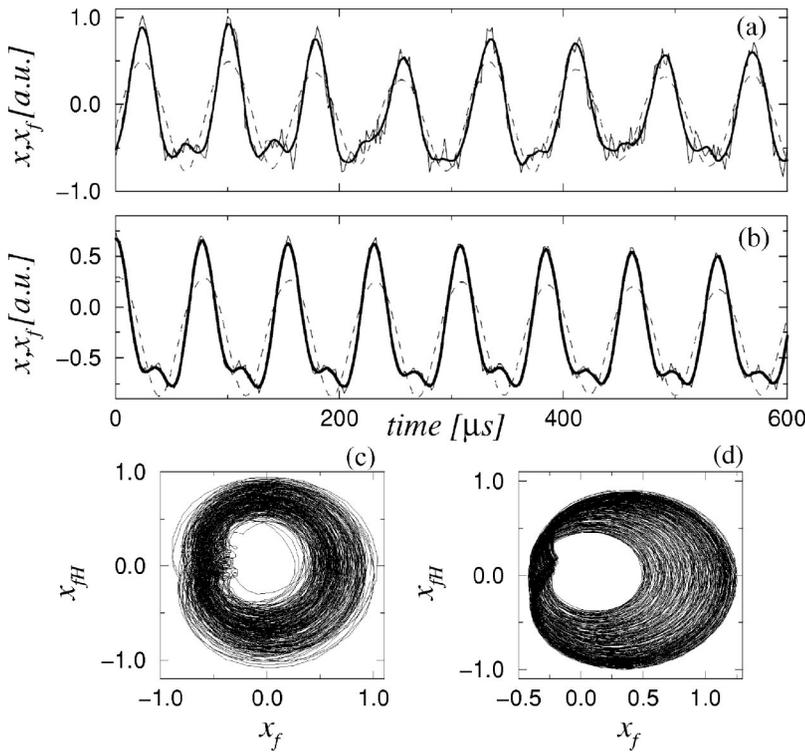


FIG. 3. (a), (b) Original (solid line) and filtered signals from two generators. The bold line corresponds to the Savitzky–Golay (4,25) filter, the dashed line corresponds to the (4,51) filter. (c), (d) Phase portraits of both oscillators in (x_f, x_{fH}) coordinates [for the Savitzky–Golay (4,25) filter].

result is $r=0.82$, while if a shorter filter (25 points) is used, the result is $r=0.4$.

We attribute this rather unexpected result—detection of bidirectional coupling in a situation when only unidirectional coupling has been imposed—to the appearance of correlations in the systems due to noise. Indeed, as one can see from the scheme Fig. 1, through the coupling elements the noise acts on the oscillator 2 as well, so in fact we have here a situation where two oscillators are driven by the common noisy force. This causes correlations in the oscillations,

moreover, a common noise can synchronize them (cf. Refs. 11, 18, and 19). Thus, here we encounter a case when the detection of the directionality of coupling is spoiled by the correlations in the signals, appearing not due to mutual interaction, but due to a common driving. These correlations may be removed by filtering, thus explaining the sensitivity of the directionality index to the filtering procedure.

An important question is how the presence of the common force can be detected just from the signals, if the structure of coupling is unknown. The answer can be obtained by

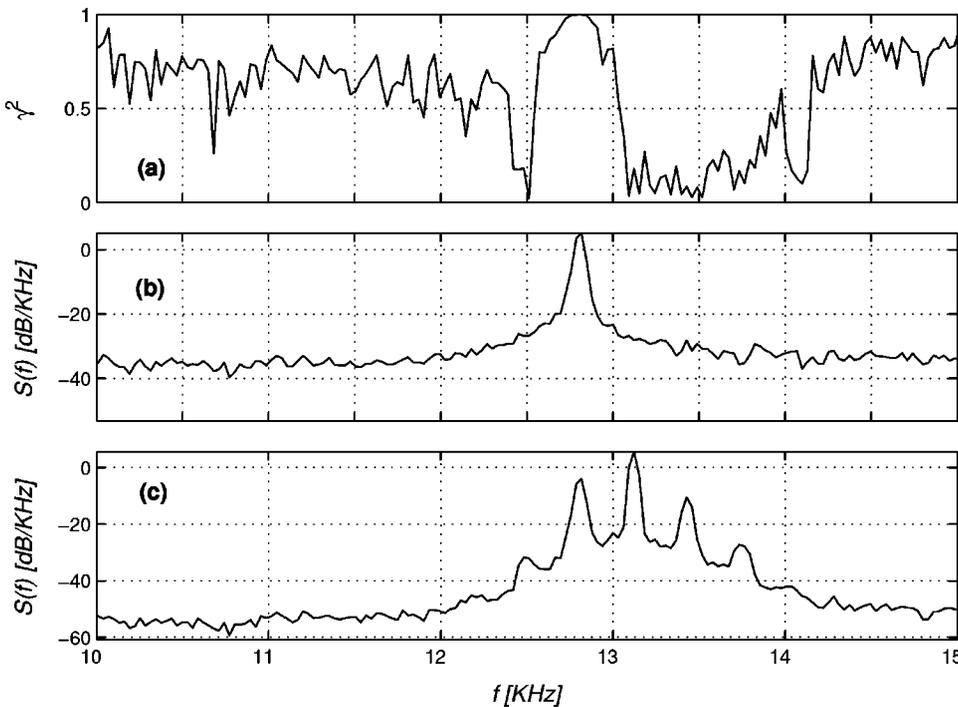


FIG. 4. (a) Coherence function γ^2 indicates correlation between the signals in a wide range of spectral frequencies. Autospectra of two generators are shown in (b) and (c).

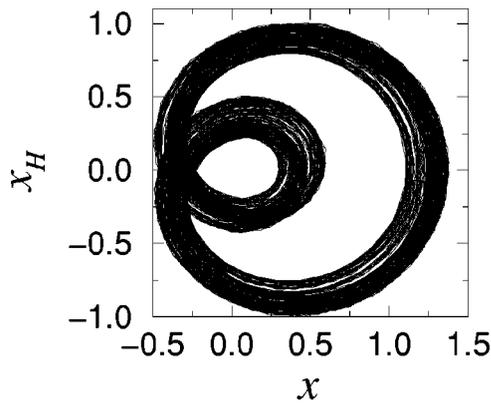


FIG. 5. Double revolution cycle (for the driven system).

performing the cross-spectral analysis of the signals and computation of the coherence function

$$\gamma^2(f) = |S_{xy}(f)|^2 / (|S_x(f)| \cdot |S_y(f)|),$$

where S_x and S_y are power spectra of signals $x(t)$ and $y(t)$, and S_{xy} is their cross-spectrum. Consider first the noise-free case. As each generator in autonomous regime is periodic, its power spectrum contains several sharp peaks, corresponding to the main frequency and harmonics. If the generator is driven by the other one, then in the power spectrum we can also see the peaks corresponding to the frequency of the driver, as well as combination of these frequencies. Naturally, the coherence function γ^2 is high at the common frequencies, but it is practically zero at other frequencies. This picture changes drastically in the presence of a wide-band common noise: in this case coherence is high in the wide band as well (Fig. 4). The same is true for the narrow-band common drive, see Ref. 15. The considered example demonstrates that if the structure of the system is unknown *a priori* then traditional cross-spectrum analysis should be performed in addition to directionality estimates.

B. Double revolution cycles

Next we consider the regime when both generators have undergone the period doubling bifurcation (Fig. 5) and the wave forms are complex. As our approach is based on the phases, and the period-doubling results mainly in the amplitude modulation, we apply the method without any changes. Alternatively, we remove running average over 25 points, which makes the amplitude of two loops nearly the same. The results are presented in Table I. One can see that the method works reasonably well: the symmetric coupling gives the least index, the unidirectional coupling the largest one, and the mixed coupling gives the intermediate index. However, for the unidirectional coupling the index is not close to one. Also in this case the quantities c_1, c_2 are relatively small, which means that the interaction can be hardly detected, which results in the relatively large error of the index estimation.

C. Chaos-chaos

Next we consider a more complicated situation of two chaotic systems (Table I). Signals are rather complex so that the phase cannot be estimated without preprocessing, because many trajectories in the (x, x_h) plane do not revolve the origin. Well-defined phases can be obtained by removing the running average over 15 points (see Fig. 6). In one case (unidirectional coupling) this does not help; in order to obtain well-defined phases, bandpass filtering (20–30 kHz) was performed.

The phases obtained from the preprocessed signals allowed us to obtain values of the directionality index that agree with the configuration of the setup. In two cases of chaos-chaos interaction we encountered another difficulty in the application of the method: the oscillations are close to synchrony. For periodic oscillators in synchrony our method fails, because in this case the phases are functionally related and one cannot obtain estimations of partial derivatives (4)

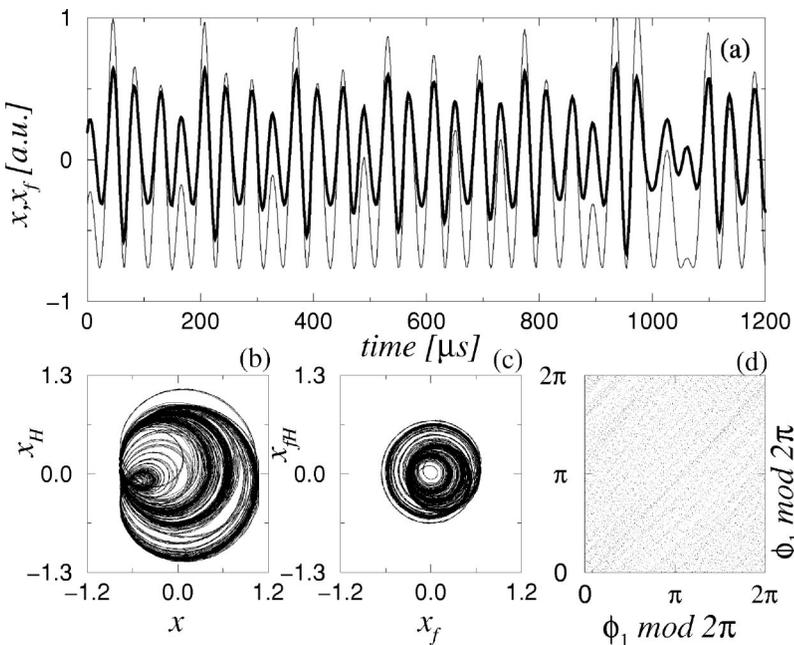


FIG. 6. (a) The original (solid line) and filtered (bold line) signals for the first oscillator. (b), (c) Phase portrait of the oscillator in (x, x_h) coordinates for the original and filtered data, respectively. (d) Phase of two systems are scattered around the (ϕ_1, ϕ_2) torus because the coupling is too weak to induce synchronization; this allows good estimation of directionality.

(the fitting procedure is singular). For chaotic oscillations near synchrony the method still works, because in this case the phases are not perfectly locked. [It corresponds to two stripes in the $\phi_2 \bmod 2\pi$ vs $\phi_1 \bmod 2\pi$ presentation, cf. Fig. 6(d).] However, the higher the degree of synchrony the less reliable are the results. As a rule of thumb, we suggest computing the synchronization index

$$\rho = \langle \cos(\phi_1 - \phi_2) \rangle^2 + \langle \sin(\phi_1 - \phi_2) \rangle^2,$$

where $\langle \rangle$ denote time averaging (see Ref. 20 for the description), and with caution interpret the results for the data sets with $\rho > 0.6$.

Certainly, one can in the same manner analyze the signals that correspond to attractors with different topologies, e.g., double revolution cycle–chaos. For brevity we do not illustrate these cases in Table I.

V. CONCLUSION

In this paper we have demonstrated that the method of detection of the direction of interaction between oscillating systems works reasonably well when applied to experimental data. The best results are obtained for simple limit cycle oscillations, but also the results for noisy oscillations, limit cycles of a complex form, and for chaotic oscillators are sound.

The main distinction of the method from other techniques consists of selecting one variable, namely the phase, for the analysis. This makes sense because the phases are mostly sensitive to the coupling. As we have shown, in several cases the extraction of the phases from the signals is nontrivial, and requires certain preprocessing. The latter can also be interesting for other cases when the phase of an irregular signal should be estimated.

We emphasize that in fact some knowledge of the system has been used in our approach: we knew that we deal with weakly interacting oscillators. In general, prior to the directionality analysis one must apply some conventional method (i.e., cross-spectral analysis) to be sure that the signals are not independent, otherwise, the question of directionality does not make any sense. Practically, it is also helpful to check whether both coefficients $c_{1,2}$ are smaller than a threshold (from our experience with simulated data we take

0.01 as the threshold value). If this is the case, the data set should be discarded. By means of cross-spectral analysis one can also exclude the case when the dependence between the signals is due to the common forcing and not due to the coupling of interacting systems (see also Ref. 15).

ACKNOWLEDGMENTS

This work was supported by the Russian Foundation of Fundamental Research, Grant No. 02-02-17578 and by U.S. Civilian Research Development Foundation for the Independent States of the Former Soviet Union, Award No REC-006.

- ¹S. J. Schiff, P. So, T. Chang, R. E. Burke, and T. Sauer, *Phys. Rev. E* **54**, 6708 (1996).
- ²M. Le van Quyen, C. Adam, M. Baulac, J. Martinerie, and F. Varela, *Brain Res.* **792**, 24 (1998).
- ³J. Arnhold, P. Grassberger, K. Lehnertz, and C. Elger, *Physica D* **134**, 419 (1999).
- ⁴R. Quian Quiroga, J. Arnhold, and P. Grassberger, *Phys. Rev. E* **61**, 5142 (2000).
- ⁵A. Schmitz, *Phys. Rev. E* **62**, 7508 (2000).
- ⁶C. Granger, *Econometrica* **37**, 424 (1969).
- ⁷T. Schreiber, *Phys. Rev. Lett.* **85**, 461 (2000).
- ⁸M. Paluš, V. Komárek, Z. Hrnčíř, and K. Štěrbová, *Phys. Rev. E* **63**, 046211 (2001).
- ⁹I. I. Blekhan, *Synchronization in Science and Technology* (Nauka, Moscow, 1981) (in Russian); English translation: ASME, New York, 1988.
- ¹⁰L. Glass, *Nature (London)* **410**, 277 (2001).
- ¹¹A. Pikovsky, M. Rosenblum, and J. Kurths, *Synchronization. A Universal Concept in Nonlinear Sciences* (Cambridge University Press, Cambridge, 2001).
- ¹²E. Mosekilde, Y. Maistrenko, and D. Postnov, *Chaotic Synchronization. Applications To Living Systems* (World Scientific, Singapore, 2002).
- ¹³M. Bennett, M. Schatz, H. Rockwood, and K. Wiesenfeld, *Proc. R. Soc. London, Ser. A* **458**, 563 (2002).
- ¹⁴M. Rosenblum and A. Pikovsky, *Phys. Rev. E* **64**, 045202 (2001).
- ¹⁵M. G. Rosenblum, L. Cimponeriu, A. Bezerianos, A. Patzak, and R. Mrowka, *Phys. Rev. E* **65**, 041909 (2002).
- ¹⁶Y. Kuramoto, *Chemical Oscillations, Waves and Turbulence* (Springer, Berlin, 1984).
- ¹⁷M. Rosenblum, A. Pikovsky, and J. Kurths, *Phys. Rev. Lett.* **76**, 1804 (1996).
- ¹⁸A. S. Pikovsky, in *Nonlinear and Turbulent Processes in Physics*, edited by R. Z. Sagdeev (Harwood Academic, Singapore, 1984), pp. 1601–1604.
- ¹⁹A. S. Pikovsky, *Radiophys. Quantum Electron.* **27**, 576 (1984).
- ²⁰M. G. Rosenblum, A. S. Pikovsky, J. Kurths, C. Schäfer, and P. A. Tass, in *Neuro-informatics*, edited by F. Moss and S. Gielen, *Handbook of Biological Physics* vol. 4 (Elsevier, Amsterdam, 2001), pp. 279–321.