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The Effect of Asymmetry upon the Fractal Properties of Synchronous Chaos in Coupled Systems with Period Doubling

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Abstract—The effect of an asymmetry upon the synchronous chaos in coupled systems with period doubling is studied by numerical methods. The introduction of an asymmetry after the loss of the transverse superstability imparts fractal properties to the synchronous chaotic attractor. On approaching the synchronization boundary (with decreasing coupling), the fractal dimension of the attractor exhibits a nonmonotonic behavior. *© 2002 MAIK "Nauka/Interperiodica".*

Investigation into the chaotic synchronization phenomenon is of significant value for both basic science and applications [1–3]. Of special interest is the study of chaotic synchronization in symmetrically-coupled identical subsystems featuring one of the classical scenarios of transition to chaos, for example, via a sequence of period-doubling bifurcations [7–13]. The regime of synchronous chaotic oscillations in such a system corresponds to a situation when the dynamic variables in both subsystems are equal. In this case, there appear two selected directions in the phase space: diagonal (in which the motions are unstable and the corresponding Lyapunov index is positive) and transverse (in which the motions are stable and the corresponding Lyapunov index is negative). Evolution of the system with variation of the nonlinearity parameter is identical to the evolution of an isolated subsystem.

The aim of this study was to determine the effect of an asymmetry (nonidentity of subsystems) on the structure of the synchronous chaotic attractor and on the evolution of this structure on approach to the synchronization boundary. For this purpose, we have numerically studied a system of dissipatively coupled quadratic mappings

$$\begin{cases} x_{n+1} = \lambda - x_n^2 + k(x_n^2 - y_n^2), \\ y_{n+1} = \lambda\delta - y_n^2 - k(x_n^2 - y_n^2), \end{cases} \tag{1}$$

where x_n and y_n are the dynamic variables, $n = 1, 2, \ldots$ is the discrete time, λ is the parameter of nonlinearity, δ is the parameter of asymmetry, and k is the coupling parameter.

For $\delta = 1$, the system is symmetric with respect to the substitution $x_n \longleftrightarrow y_n$. For this case, the system dynamics was studied in sufficient detail [2, 12–15]. For $k > 0$ in the region above the critical level ($\lambda > \lambda_c$, where $\lambda_c = 1.40115518909\ldots$ is the critical value for the isolated subsystems [16]), system (1) exhibits a synchronous chaotic regime with the phase portrait situated on the diagonal of the (x_n, y_n) plane. In this case, behavior of the coupled systems is equivalent to dynamics of an isolated subsystem. Estimation of the correlation dimension of a chaotic attractor for the critical value of the parameter ($\lambda = \lambda_c$) yields $d_c = 0.54$ (we have calculated a reduced correlation dimension by using 40000 values and 5000 reference points determined to within 10^{-18}), which is close to the Hausdorff dimension of the critical attractor of a quadratic mapping [16, 17]. As the parameter λ grows, the dimension of the synchronous attractor increases and becomes equal to unity. This is related to the fact that the dimension of the chaotic attractor for the quadratic mapping is unity and the synchronous attractor of a coupled system is situated on a straight line—the diagonal $x_n = y_n$ in the phase plane.

When $\delta \neq 1$, the symmetry is broken but the synchronization regime is retained. Figure 1a shows the structure of the plane of parameters (k, λ) for $\delta = 0.97$. Here, nonshaded areas correspond to periodic synchronized cycles, light-gray areas represent synchronous chaotic regimes, dark-gray areas correspond to nonsynchronized regimes, solid lines indicate the period-doubling bifurcations, and figures at the lines indicate the cycle periods. Figure 1b presents the phase portraits of chaotic attractors and shows some fragments in more detail.

Let us consider evolution of the cycle with period 1 when the parameter λ increases at constant values of $k = 0.5$ and $\delta = 0.97$. The growth of λ is accompanied by a sequence of period-doubling bifurcations, which terminates (at $\lambda \approx 1.4229918\ldots$) by the transition to a synchronous chaos (Fig. 1b, fragment 1). For $k = 1$, the determinant of the linearization matrix equals zero and, hence, all cycles (both stable and saddle-point ones) of system (1) are superstable in the transverse direction. In this case, the critical attractor is situated on the straight line and the correlation dimension is estimated at $d_c = 0.5$ (which is close to a correlation dimension of the critical attractor of a one-dimensional quadratic mapping calculated in [16, 17]. As the parameter λ grows further. The connectivity of the synchronous attractor gradually decreases (see Fig. 1b, where fragment 2 shows a simply connected character), while the correlation dimension increases to reach $d_c = 1$. Thus, in the case of an asymmetry introduced in a system with the transverse superstability, fractal properties of the synchronous chaos are analogous to those in the symmetric case.

As the coupling parameter decreases, the in-phase stable and saddle-point cycles of system (1) lose superstability in the transverse direction, while the unstable manifolds of their saddle cycles are no longer situated on the same straight line. Figure 1b (fragment 3) illustrates the case of a critical attractor for $k = 0.4$. With increasing λ, each element of the critical attractor evolves in its own direction in the phase space and, as a result, the critical attractor is not situated on a straight line and retains a fractal character (fig. 1b, fragment 4).

Figure 2a shows dependence of the correlation dimension of synchronous chaos on the coupling parameter for $\lambda = 2$. Figure 2b presents the phase portraits of attractors and shows some fragments in more detail. As the λ value decreases, the dimension grows above unity and the attractor is no longer situated on the diagonal of the phase plane. A greater scale (Fig. 1b, fragment 4) reveals a complex fractal structure representing an infinite system of lines. As the coupling parameter decreases further, the attractor expands in the transverse direction (Fig. 2b, fragment 1) and increases in dimension. The line structure of the synchronous attractor is revealed on a smaller scale. One could reasonably suggest that, on approaching the synchronization boundary, the attractor dimension (as well as the second Lyapunov index) would monotonically grow.

However, when the coupling parameter k approaches 0.25, the synchronous attractor begins to contract in the transverse direction (while the second Lyapunov index keeps growing) and the attractor dimension decreases to become close to unity (although the attractor is still not situated on a straight line (Fig. 2b, fragment 2). It should be noted that, in the vicinity of the point at $\lambda = 2$ and $k = 0.25$, the system features period-doubling bifurcations of the saddle-point cycles embedded into the synchronous chaotic attractor. Therefore, the contraction may be caused by a change in configuration of the manifolds of unstable cycles. Further decrease in the coupling parameter leads to an increase in the correlation dimension and to the attractor expansion in the transverse direction (Fig. 2b, fragments 3 and 4). On approaching the boundary of the synchronization region, the attractor keeps expanding, while its dimension decreases again.

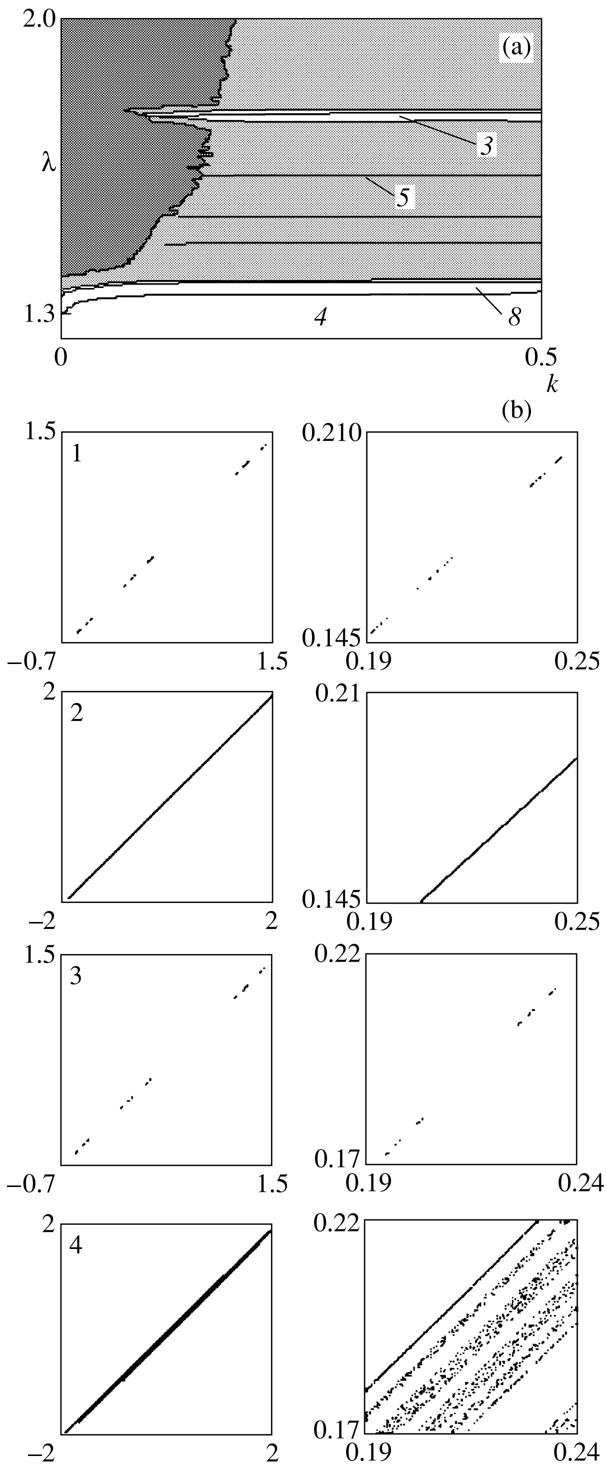

Fig. 1. Diagrams showing (a) the structure of the plane of parameters (k, λ) for $\delta = 0.97$ and (b) the phase portraits and their fragments for (*1*) $\lambda = 1.4229918\ldots$, $k = 0.5$; (*2*) $\lambda = 2$, $k = 0.5$; (*3*) $\lambda = 1.4229918\ldots$, $k = 0.4$; and (*4*) $\lambda = 2$, $k = 0.4$.

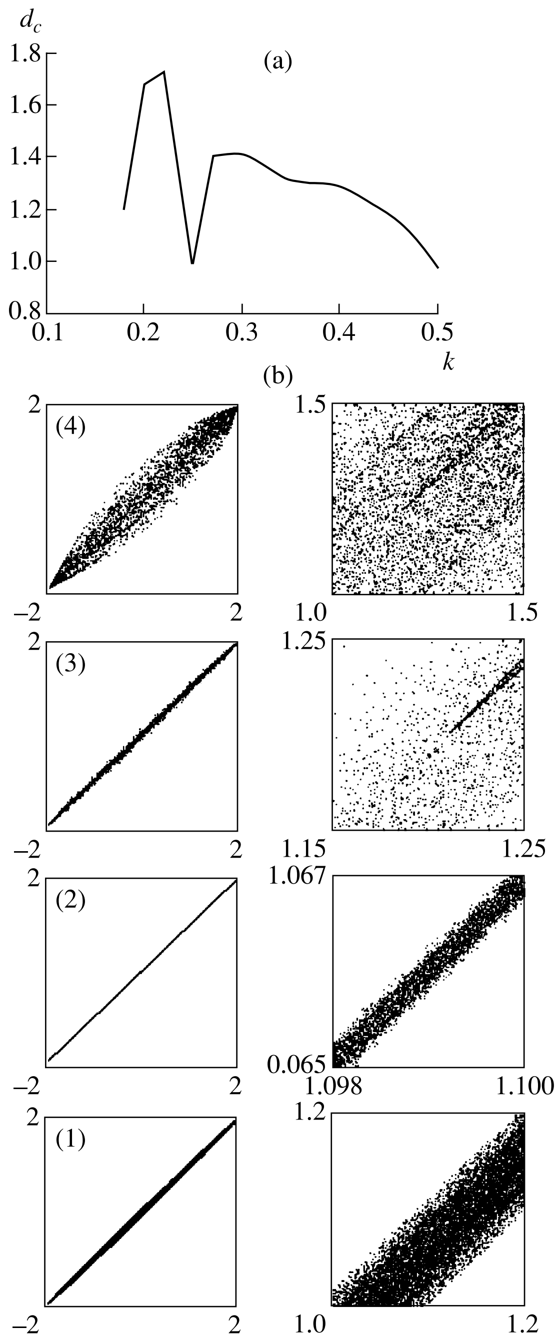

Fig. 2. Diagrams showing (a) the plot of correlation dimension d_c versus parameters k for $\lambda = 2$ and (b) the phase portraits and their fragments for $k = 0.3$ (1), 0.25 (2), 0.24 (3), and 0.225 (4).

Thus, we can draw the following conclusions. The introduction of an asymmetry (nonidentity of subsystems) into symmetrically coupled systems does not influence the dimension of the synchronous chaotic attractor (the attractor dimension characteristics remain the same as those in an isolated subsystem). When the superstability is lost, the attractor dimension begins to increase and exhibits a nonmonotonic behavior on approaching the synchronization boundary.

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