Two-level control of chaos in nonlinear oscillators

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Control of chaos was achieved experimentally for the first time in a nonautonomous RL-diode circuit using a two-level system and modeled numerically using a multiparameter one-dimensional mapping). This system is a modification of the classical Ott–Grebogi–Yorke method but is distinguished by its ease of implementation in real systems. © *1999 American Institute of Physics.* [S1063-7850(99)02702-0]

1. The experimental implementation of methods of controlling chaos in nonlinear systems frequently comes up against the complexity of the algorithms for the variation of the control parameter and requires the development of simpler variants. Such approaches may include the modification of the Ott–Grebogi–Yorke method[1] proposed in Ref. 2, in which the variation of the parameter p, which in the classical variant is proportional to the deviation of the state of the system from that being stabilized, is replaced by switching between two fixed values p and p_2, i.e., by the two-level equation

$$p=p_0+k\,\mathrm{sgn}(x_n-x_0)=\begin{cases}p_1, & \text{if } x_n\le x_0\\ p_2, & \text{if } x_n>x_0,\end{cases} \tag{1}$$

where $k=(p_2-p_1)/2$, $p_0=(p_1+p_2)/2$, $p_2>p_1$, and x_n and x_0 are the instantaneous value of the variable and the value on the stabilized unstable orbit in the Poincaré cross section (in general a vector).

Here two-level stabilization is achieved in physical and numerical experiments in two variants: (a) with the algorithm (1) "switched on" when the mapping point in the phase space reaches a given vicinity of the stabilized orbit (on entering a "window") and (b) in a simpler variant under the continuous action of the algorithm (1) without introducing a window. The objects of the investigation are a nonlinear dissipative oscillator periodically excited by an external force, i.e., a nonautonomous RL diode circuit (shown by the heavy line in Fig. 1) and a multiparameter one-dimensional mapping which accurately models the complex dynamics of the experimental system in the subharmonic resonance frequency range:

$$x_{n+1}=A+x_n\exp(-d/N)\cos(2\pi/(N(1+\beta x_n))), \tag{2}$$

where A is the analog of the amplitude of the external action, $N=T_0/T$ is the normalized frequency of the action, d characterizes the dissipation, β is the nonlinearity parameter, and $n=1,2,3\ldots$ is the discrete time.[1)]

We demonstrate the efficiency of this simplified control procedure and analyze its capabilities and shortcomings: the motion takes place in a given range of the dynamic variable near a selected orbit; an increase in the period of the stabilized motion is accompanied by a decrease in the range of parameters in which stabilization is possible.

2. We shall first analyze the two-level control procedure using the multiparameter mapping (2), which we rewrite in the form $x_{n+1}=A+f(x_n)$, where A is the control parameter. In the immediate vicinity of the fixed point x_0, the dynamics of the system are described by

$$x_0+\tilde{x}_{n+1}=A_0+f(x_0)+\left.\frac{df}{dx_n}\right|_{x_0}\tilde{x}_n+\tilde{A}_n, \tag{3}$$

where $\tilde{x}_{n+1}$ and $\tilde{x}_n$ correspond to small perturbations and the variation of the control parameter $\tilde{A}_n$ is also introduced for control purposes. We separate Eq. (3) into the equation for the fixed point

$$x_0=A_0+f(x_0) \tag{4}$$

and the equation for the variations

$$\tilde{x}_{n+1}=\tilde{A}_n+\left.\frac{df}{dx_n}\right|_{x_0}\tilde{x}_n \tag{5}$$

We set ourselves the task of using $\tilde{A}_n$ to restrict the motion to the immediate vicinity of the unstable fixed point x_0 (an unstable cycle of period 1) for values of the parameters A_0, β, d, and N corresponding to chaos. For this purpose we determine the value of x_0 from Eq. (4), find the derivative of the function f at this point, and define the window σ. We then iterate Eq. (3) beginning with various initial conditions until x_n is within σ of the point x_0, i.e., $|\tilde{x}_n|<\sigma$. The parameter $\tilde{A}_n$ should then be varied in accordance with rule (1) ($\tilde{A}_n=k$ for $x_n>0$; $\tilde{A}_n=-k$ for $x_n<0$), so that the mapping point does not leave the given vicinity of the fixed point, $|\tilde{x}_{n+1}|<\sigma$. This is possible if, in accordance with Eq. (5), the following inequality is satisfied:

$$-\sigma\left(1+\left.\frac{df}{dx_n}\right|_{x_0}\right)<k<\sigma. \tag{6}$$

Thus, the condition for finding x_n near a fixed point of period 1 is $(df/dx_n)_{x_0}>-2$. This series of actions is easily trans-

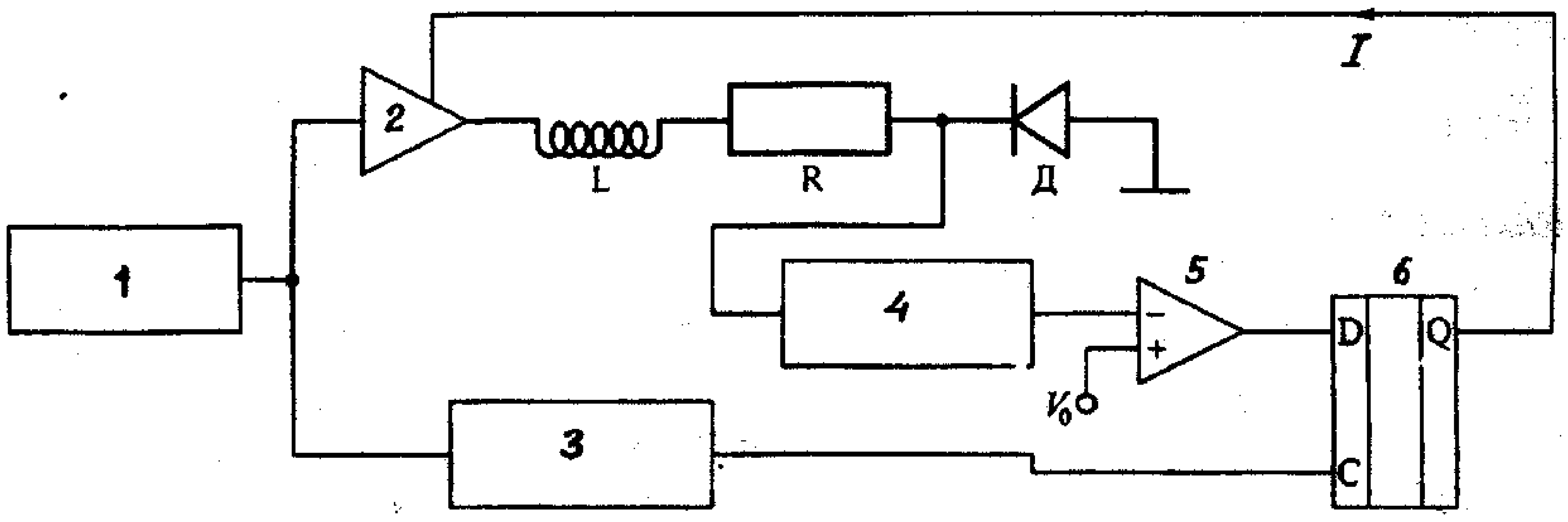

FIG. 1. Schematic of experiment (the heavy line indicates the system being studied): *1* — oscillator, *2* — amplifier, *3* — pulse shaper, *4* — amplitude detector, *5* — comparator, *6* — trigger, and *I* — control circuit.

ferred to cycles of period $m=2,4,8\ldots$, if we use formulas for the corresponding iterations $x_{n+m}=f^m(x_n)$ rather than Eq. (2).

The results of numerical experiments to implement this control procedure using the mapping (2) for values of A_0, β, d, and N corresponding to chaos are plotted in Fig. 2. With no control the steady-state motion of the system on the phase plane takes place on a chaotic attractor occupying an extended region of the parabola (Fig. 2a), but when the control is switched on it is confined to a given vicinity of the point x_0. The degree of "compression" of the attractor and the duration of the establishment process (between switching on the control and stabilization) is determined by the choice of k: for small k the values of x_n are positioned near x_0, as k increases the spread of x_n increases and the duration of the transition process decreases. This is illustrated by the time series of the oscillations in the system shown in Figs. 2b and 2c: for $k=0.05$ stabilization occurs after 80 iterations, whereas for $k=0.1$ it occurs after around 20. The calculations show that two-level control can also stabilize the motion near unstable cycles of period 2 and 4. However, the higher the period of the unstable cycle, the larger the factor multiplying the absolute value and the smaller the region in which stabilization can be achieved.

We shall simplify the control procedure by retaining two fixed values A_0+k and A_0-k after the parameter A but removing the condition that x_n must lie within σ of the fixed point of period 1. In this case, the control parameter A does not have values of A_0 as in the previous case. The calculations show that the new procedure also stabilizes the motion near the fixed point of period 1 but appreciably simplifies the experimental implementation of the control system.

3. In a physical experiment, the two-level control scheme (Fig. 1) was constructed in accordance with the second (simplified) variant of those considered in Sec. 2. An RL circuit with a diode ($L=100$ mH, KD202 diode) was excited by a pulsed signal from a generator via an amplifier. The gain p could have two values: $p_1=1$ and $p_2=1+\Delta$, where Δ was varied between 0.00 and 0.07 during the experiment. The control system was used to compare the voltage V at the diode and at the reference voltage source V_0 at times when the external action has a certain phase. Depending on the sign of $V-V_0$ the level of the exciting signal was set at one of two values. The form of the stabilized motion was determined by defining the reference voltage V_0, the value of Δ, and the time interval between the comparisons of the diode and reference voltage. Note that as in Ref. 2, there are two levels of the control parameter p_1 and p_2 in the experimental procedure, but the control parameter never has the value $p_0=(p_2+p_1)/2$.

The results of the experimental investigations showed that the simplified two-level control variant is effective and revealed qualitative agreement with the results of the numerical modeling. Figure 2d illustrates the stabilization of the motion near an unstable cycle of the period of the external action for the case, where before control was switched on chaotic oscillations existed in the system, formed as a result of a series of doubling bifurcations of this cycle.[2)] This figure

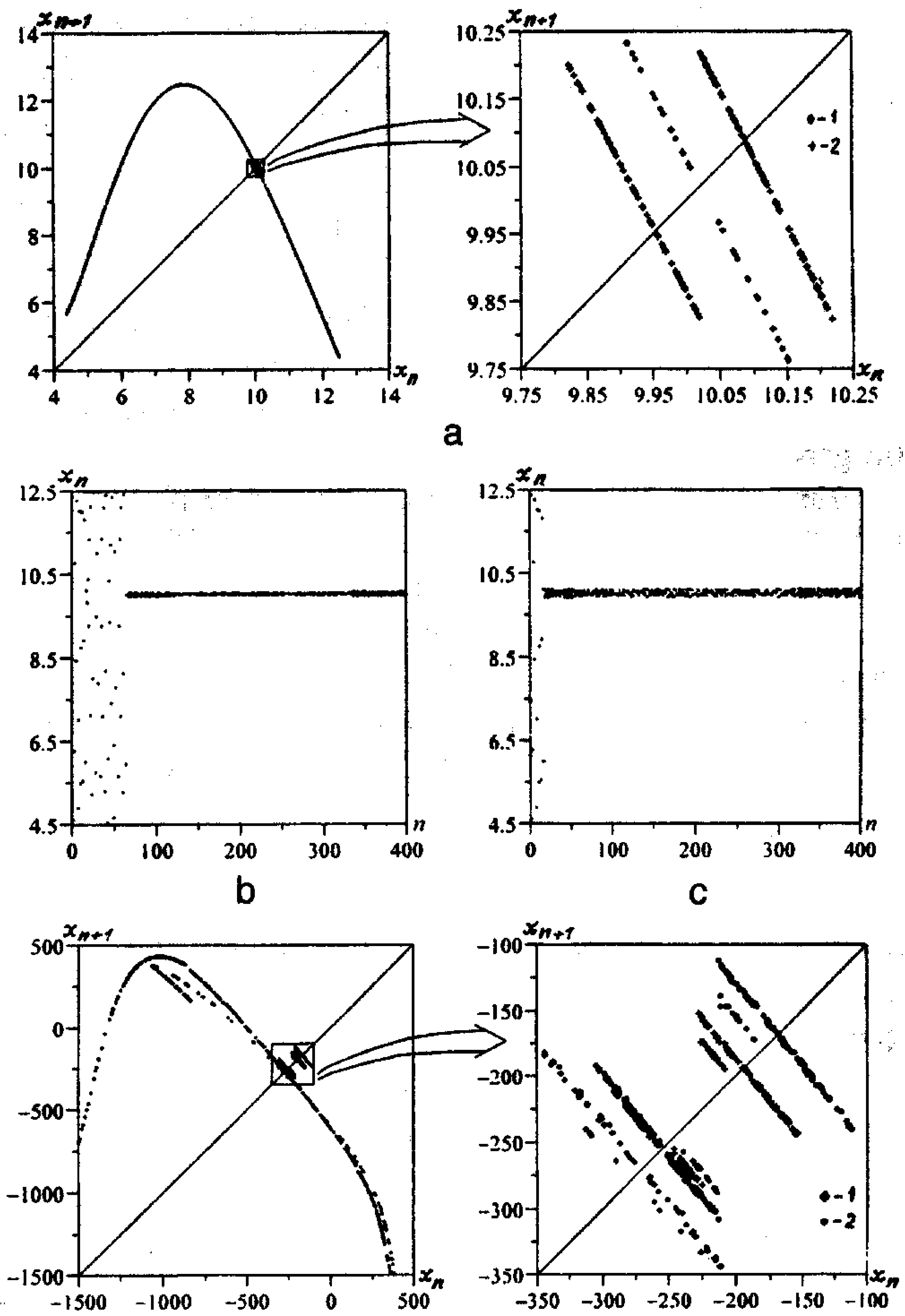

FIG. 2. a — Mapping on the plane x_{n+1}–x_n with control switched off (*1*) and switched on (*2*); b, c — time series of oscillations in the system (2) for $A_0=7$, $d=0.13$, $\beta=0.205$, $N=0.4$ for the same initial conditions for $k=0.05$ and $k=0.10$, respectively; d — mapping of the sequence of the experimental system without control (*1*) and with control for $\Delta=0.04$(*1*) and $\Delta=0.07$ (*2*).

gives the dependence $V_{i+1}(V_i)$, where V_i is the voltage at the diode at discrete times i after the period of action. The main diagram shows the case where the control system is switched off and the magnified fragment gives the dependence after switching on the control for various values of Δ (crosses denote $\Delta = 0.04$ and circles $\Delta = 0.07$). As in the numerical experiment, when the control is switched on, the points in the stroboscopic cross section form piecewise linear sets with a discontinuity near the reference value. As Δ decreases, the vicinity of the unstable cycle visited by the mapping point becomes shorter. As Δ approaches zero, control is abruptly terminated.

4. A two-level control system can organize the motion of the system in a given range of the phase space of nonautonomous oscillators in chaotic motion based on any of the subharmonic cycles. The method is fairly approximate and is stable with respect to the unavoidable perturbations in a physical experiment. An advantage of this control system is that the algorithm and thus the design of the control circuits is extremely simple. However, it can only confine the motion in a given interval and is not suitable for prolonged motion on an unstable orbit, as in the classical stabilization procedure.

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[1)]An *RL* diode circuit is widely used as a selective element with electronic tuning, as a frequency divider and multiplier, and has been considered as a memory cell with phase recording of information,[5] but recently, following the observation of chaotic dynamics[3] in this circuit, it has become a test bed for studying various nonlinear oscillatory phenomena. The model mapping (2) was obtained for dissipative oscillators with "soft spring" nonlinearity, periodically excited by forcing pulses during which additional losses are introduced into the system. For an *RL* diode circuit this is achieved when the pulses are positive for the diode.[4]

[2)]The thickening of the lines of the experimental mapping and their separation in Fig. 2d is caused by the real system not being one-dimensional and by technical factors (the unavoidable propagation of the control signal of the electronic switch into the excitation signal of the oscillator circuit.

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