Controlling spatiotemporal chaos in a chain of bistable oscillators

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The processes in coupled oscillators and their control under bistability conditions are considered. The possibility of stabilizing spatially homogeneous states is demonstrated. © *1999 American Institute of Physics.* [S1063-7850(99)02406-4]

1. The processes in spatially distributed (multidimensional) oscillatory systems are often simulated by oscillators coupled to one another in a chain or an array. Such ensembles have also been used successfully as models of distributed media. A chain of identical dissipative oscillators closed in a ring, in which the oscillators are subjected to cophasal excitation by a periodic external force, can be used to analyze the possibility of controlling chaos. The elements of the chain are nonlinear, capable of undergoing regular and chaotic oscillations, and bistable. The last property means that two different forms of steady motions can take place at fixed values of the parameters. Bistability is typical of nonautonomous oscillatory systems at nonlinear resonance, where hysteresis is observed when the parameters are varied. The coupling between the elements of the chain is symmetric and local (the elements interact only with neighbors), as well as diffusive (dissipative). The problem of controlling spatiotemporal chaos is considered in its classical form, i.e., stabilization of motions in an unstable limit cycle embedded in a chaotic attractor by small changes in a controlling parameter.[1–5] The possibility of stabilizing spatially homogeneous states of an ensemble of bistable elements using an element-by-element regulation procedure[6] and the effectiveness of this approach in the presence of noise are demonstrated.

2. The following discrete model was investigated under the condition that the elements in the chain are identical and their excitation is cophasal:

$$x_{n+1}^m = (1-2k) f(x_n^m) + k[f(x_n^{m+1}) + f(x_n^{m-1})], \tag{1}$$

where x is a dynamic variable, $n=0,1,2,\ldots$ is the discrete time, m is the number of the element in the chain, and k is the coupling coefficient. The boundary conditions are periodic: $x_n^1 = x_n^{M+1}$, where M is the number of elements in the chain. The basic chain element $f(x_n^m)$ is a mapping, which reflects the temporal dynamics of an oscillator. We used the multimodal multiparameter mapping

$$x_{n+1} = f(x_n) = x_n \exp[-d/N] \cos\left[\frac{2\pi}{N(1+\beta x_n)}\right] + A, \tag{2}$$

whose parameters characterize the amplitude (A) and normalized frequency (N) of the external periodic disturbance, the dissipation (d), and the nonlinearity (β). The mapping (2) has both regular and chaotic solutions, qualitatively describes the temporal dynamics of a nonlinear dissipative oscillator and the structure of its bifurcation sequences in the region of the existence of subharmonic oscillations and their evolution to chaos, as well as reflects such nonlinear phenomena as hysteresis, bistability, and multistability.[7,8] The form of a steady multistable oscillatory state is specified by the initial conditions.

At certain values of the parameters the system of equations (1) and (2) goes over to a regime of developed spatiotemporal chaos.[9,10] The attractors corresponding to such states contain a set of saddle periodic orbits, which can be stabilized by a controlling disturbance acting on the parameters of the system.

3. Spatially homogeneous states of the chain were stabilized for two typical cases: for values of the parameters corresponding to an absence of hysteresis and the associated bistability in the elements of the chain and for the presence of bistability in a single element. In accordance with the control procedure used,[6] the controlling disturbance is imposed on the parameter A of each of the coupled oscillators. Thus, the parameter A depends both on the moment in time and on the number of the element m and can be written in the form

$$A = A_n^m = A_0 + \tilde{A}_n^m, \tag{3}$$

where A_0 is the constant component and $\tilde{A}_n^m$ is the variable component. Each of the elements in the chain is successively subjected to the control procedure as its dynamic variable x_n^m comes into the vicinity of the state being stabilized.

The states stabilized in our work are fixed points $\bar{x}$ of the mapping (2). Then, when x_n^m enters a small vicinity of $\bar{x}$, we can write

$$x_{n+1}^m = \bar{x} + \tilde{x}_{n+1}^m, \quad x_n^m = \bar{x} + \tilde{x}_n^m, \tag{4}$$

where $\tilde{x}_{n+1}^m$ and $\tilde{x}_n^m$ are small perturbations. Substituting (3) and (4) into (1) and linearizing the expression obtained, we obtain the equation for a fixed point

$$\bar{x} = A_0 + \bar{x} \exp[-d/N] \cos\left[\frac{2\pi}{N(1+\beta\bar{x})}\right] \tag{5}$$

and a linearized equation for the perturbations $\tilde{x}_{n+1}^m$, from which we find the value $\tilde{A}_n^m$, at which the perturbation $\tilde{x}_{n+1}^m$ becomes equal to zero:

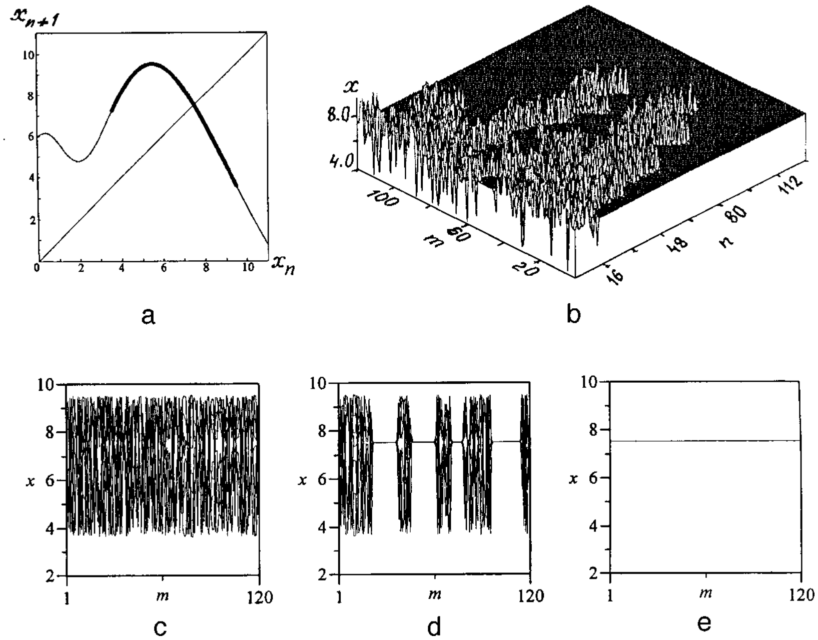

FIG. 1. a — Plot of the mapping (2) for $A=6$, $d=0.2$, $N=0.5$, and $\beta=0.2$ (the chaotic attractor isindicated by the thick line); b — spatiotemporal diagram of the establishment of a homogeneous regime of period 1; c — regime of developed spatiotemporal chaos; d–e — controlled transition to a spatially homogeneous regime of period 1 (e); d — intermediate stage of the transition.

$$\tilde{A}_n^m = -\exp[-d/N]\Bigg\{(1-2k)(x_n^m-\overline{x}) \times\left(\overline{x}\sin\left[\frac{2\pi}{N(1+\beta\overline{x})}\right]\frac{2\pi\beta}{N(1+\beta\overline{x})^2} +\cos\left[\frac{2\pi}{N(1+\beta\overline{x})}\right]\right)+k\left(x_n^{m+1}\cos\left[\frac{2\pi}{N(1+\beta x_n^{m+1})}\right] +x_n^{m-v}\cos\left[\frac{2\pi}{N(1+\beta x_n^{m-1})}\right] -2\overline{x}\cos\left[\frac{2\pi}{N(1+\beta\overline{x})}\right]\right)\Bigg\}. \quad (6)$$

The controlling disturbance acts when two conditions are satisfied simultaneously, viz., $|x_n^m-\overline{x}|<\varepsilon$ and

$$\left|k\left\{\left(x_n^{m+1}\cos\left[\frac{2\pi}{N(1+\beta x_n^{m+1})}\right]-\overline{x}\cos\left[\frac{2\pi}{N\left(1+\beta\overline{x}\right)}\right]\right) +\left(x_n^{m-1}\cos\left[\frac{2\pi}{N(1+\beta x_n^{m-1})}\right] -\overline{x}\cos\left[\frac{2\pi}{N(1+\beta\overline{x})}\right]\right)\right\}\right|<\varepsilon,$$

i.e., when both terms in (6) are relatively small.

4. Let us first examine the controlled transition from a regime of spatiotemporal chaos to a spatially homogeneous regime for values of the parameters at which the chain elements have a single fixed point of period 1. The plot of the function (2) for such values of the parameters and the chaotic attractor which exists for them are shown in Fig. 1a. The single orbit of period 1 appearing in the chaotic attractor with these values of the parameters (Fig. 1) can be stabilized by a controlling disturbance of the form (6) using the scheme described above. The magnitude of the controlling disturbance decreases with time. We note that the stabilization of regimes with time and in space takes place only when there is weak coupling between the elements of the chain. For example, $k=0.005$ for the plots in Figs. 1c–1e. This is because the algorithm used is based on a method of successive stabilization of the elements in the chain, and the weaker is the coupling between the elements, the longer will the element subjected to the controlling disturbance reside in the vicinity of the unstable state being stabilized and, consequently, the higher will be the probability of cluster formation.

Let us now consider the stabilization of spatially homogeneous states of period 1 for the second typical case, in which the parameters of the system are such that bistability is observed in the elements of the chain and two chaotic attractors coexist in them. The plot of the mapping (2) corresponding to this situation and the forms of the chaotic attractors are presented in Fig. 2a. For the values of the parameters chosen the system has three unstable fixed points, two of which appear in a chaotic attractor. If the initial conditions in all the elements of the chain are assigned in the basin of attraction of one chaotic attractor, the situation is similar to the one considered above for the absence of bistability. Then, each orbit of period 1 appearing in the attractor is stabilized in the chain according to the scheme described. If the initial conditions are such that some of the elements in the chain undergo oscillations in one chaotic attractor and others do so in the other chaotic attractor, a fairly broad window with respect to ε and, therefore, a larger value of the controlling disturbance are required to stabilize any of the regimes of period 1. This condition is fundamental, since regardless of the strength of the coupling between the elements the oscillations of an individual oscillator can occur in the absence of the controlling disturbance in only one of the attractors. Therefore, stabilization of the fixed point appearing in the

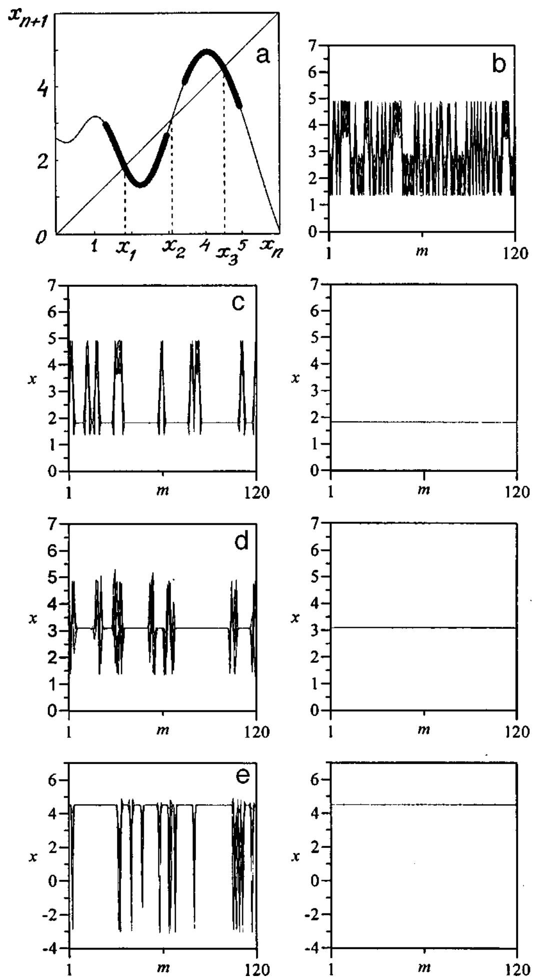

FIG. 2. a — Plot of the mapping (2) for $A=2.6$, $d=0.15$, $N=0.28$, and $\beta=0.2$ (the coexisting chaotic attractors are indicated by the thick lines); b — regime of developed spatiotemporal chaos in a region of bistability; c–e — controlled transition to spatially homogeneous regimes of period 1 (right-hand column) with various oscillation amplitudes: c — $x=x_1$, d — $x=x_2$, e — $x=x_3$ (left-handcolumn — intermediate stage of the transition).

other attractor is possible only for a value of ε greater than the interval with respect to x between the boundaries of the chaotic attractors. The results demonstrating the controlled transition to each of the three possible spatially homogeneous regimes of period 1, including the one which does not belong to either of the attractors, are shown in Fig. 2.

The magnitude of the controlling disturbance needed to bring the chain into a spatially homogeneous regime in the bistability region can be reduced significantly, if random noise is allowed to act on the system. Owing to the presence of noise, switching between the bistable states becomes possible, and the oscillations of an individual oscillator can take place alternately near each chaotic attractor. By supplying noise to the system and simultaneously applying a controlling signal to it, we can try to force the oscillations of all the elements of the chain to move into the vicinity of only one selected chaotic attractor. Thereafter, the noise can be removed, and stabilization of the spatially homogeneous regime of period 1 appearing in that attractor can easily be achieved. For the case depicted in Fig. 2b the application of random noise with a maximum amplitude having an absolute value $\Delta=0.2$ to the system led to fourfold decreases in the absolute values of the controlling signal and the interval ε in which control takes place.

5. The approach used can be applied to the stabilization of spatially homogeneous and spatially periodic regimes with different temporal and spatial periods, as well as to the stabilization of spatial regimes in a two-dimensional array of bistable oscillators.

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