Experimental confirmation of universality and scaling laws for a model oscillator with delayed feedback

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Systems with deleyed feedbac are being studied extensively in radiophysics and electronics, nonlinear optics, and certain other fields [1-7]. One of the current problems in the dynamics of systems of this class is that of determining the specific features of their behavior near the threshold for the onset of chaos. Systems with a delay should be thought of as distributed systems in which the role of spatial structures is played by the configurations of the signal which are presented during the delay interval. Despite several experimental and numerical studies on certain aspects of the transition to chaos in systems with delay, the problem remains largely unresolved. An interesting direction in this research is an analysis oriented toward the concepts of universality and similarity (scaling), which was mentioned in Ref. 8 in connection with systems with delay. For an experimental test of the conclusions which follow from the results of that study, a special physical model consisting of a system with a digital delay 1ine has been developed. This delay line can introduce a delay which can be varied over a wide range. We will compare the experimental results with the results of a numerical solution of the differential equations which describe the system and also with the conclusions which follow from the universality and scaling relations found In Ref 8.

The system studied in the present experiments consists of a chain of the following elements, which has been closed into a ring:

1. A nonlinear element, which has an essentially instantaneous response over the time scale of interest and which has a nonlinear characteristic with a quadratic extremum. This element was fabricated as a transistor circuit. The output voltage can be approximated very accurately as a function of the input voltage by the expression

$$f(U) = \lambda - \ln\left[e^{2U-11.18} + e^{-4.5U-0.36} + 0.00115\right]. \quad (1)$$

The quantity λ can be adjusted smoothly and will play the role of the basic parameter controlling the transition of the system to chaos.

2. A low-frequency filter, which consists of a chain of N = 6 identical RC circuits which are connected in series through buffer amplifiers (which have a gain unity). The pulsed response function of this filter is $\dfrac{t^{N-1}(RC)^{-N}}{(N-1)!} \cdot e^{-t/RC}$, and its Fourier transform (the transfer function) is S $=(1+i\omega RC)^{-N}$. Approximating the logarithm in the transfer function by the first terms of a Taylor Series, we find $S \approx e^{-i\omega\Delta T - \omega^2\tau^2}$.

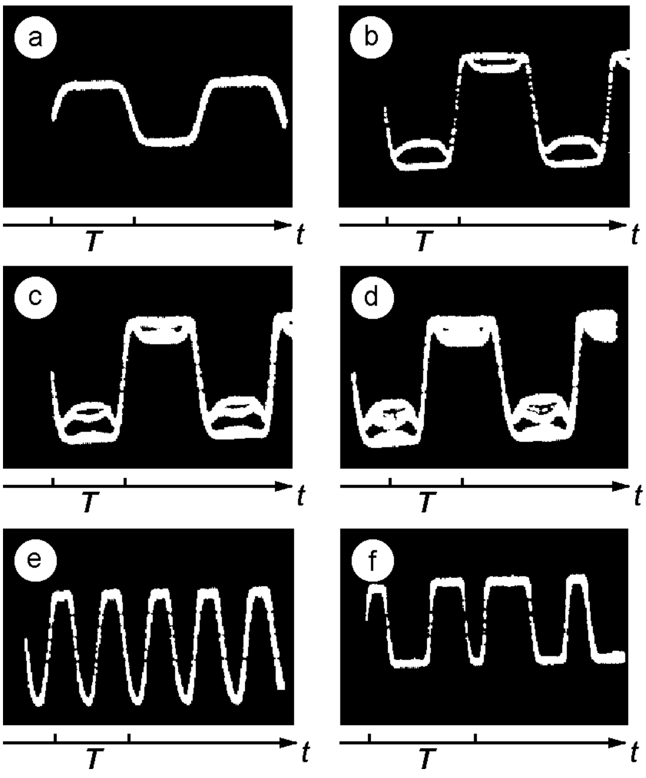

FIG.1. Oscilloscope traces of the oscillations in the case in which the sweep time of the oscilloscope is approximately $3T$. $T/\tau \approx 23$ (a-e), $T/\tau \approx 34$ (f)

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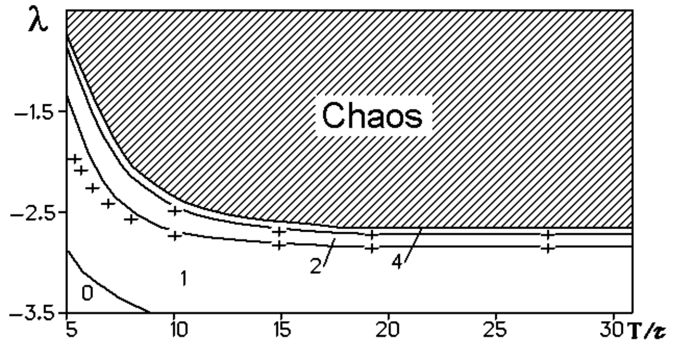

FIG.2. Partitioning of the (λ, T/τ) parameter plane into characteristic regimes. The solid lines show experimental values of λ. 1 – values found through the use of universal function and constants of Ref.[8]; 2 – results of numerical studies of system of differential equations (2). The numerals specify the oscillation period, in units of T.

The quantity $\Delta T = NRC$ characterizes the delay introduced by the filter, while $\tau = \sqrt{NRC/2}$ characterizes the duration of the response to a pulsed perturbation, i.e., the inertial properties of the chain. The response function of the filter is approximately symmetric with respect to the point t = ΔT, in accordance with the assumption in Ref. 8.

3. The delay line. The signal which is received at the input is transformed to digital form and stored in on-line memory, which introduces the delay. At the output, the signal is transformed back into analog form. By controlling the memory, one can vary the delay time T_0 over three orders of magnitude while keeping the other characteristics of the circuit constant (the input and output impedance R, the transfer ratio, etc.).

The overall dynamics of this ring chain is described by the following system of differential equations:

$$RC\frac{dU_i}{dt} + U_i = \begin{cases} f(U_N(t-T_0),\ i=1 \\ U_{i-1},\ i=2,...,N, \end{cases} \qquad (2)$$

where U is the voltage at the output of the i-th filter element.

We fix the normalized delay T_0/τ , and we examine the changes in the nature of the dynamic regimes as the control parameter λ is increased. At a certain instant, self-oscillations appear in the system, with a period which is highly accurately equal to $2T$, where $T = T_0 + \Delta T$ is the time required for the signal to pass through the feedback loop, with the delay introduced by the delay line and the filter taken into account. If the quantity T/τ is not too small, the signal rapidly acquires, as the parameter λ increased, a characteristic shape, which is approximately rectangular (Fig. la). The typical width of the switching fronts (drops) is determined by the response time τ, while the length of the gently sloping regions is determined by the delay time T. With a further increase in λ, we observe a sequence of period doublings and then a transition to chaos (b-d In Fig. 1). In the numerical calculations, one can detect a large number of doubling bifurcations within the framework of Eq. (2), while in the experiments only two can be seen reliably, because of the rather high level of digitization noise. Nevertheless, in the experiments one can clearly follow the features of the evolution of the structures on the route to chaos which are characteristic of a distributed system and was considered in Refs. 8 and 9. Near the drops at the edges of the gently sloping regions, the signal has some irregularities or "tails," which penetrate somewhat into the gently sloping regions. It can be seen from Fig. 1 that the tails which arise in the course of one of the period doublings are longer than those which existed earlier. This observation agrees with the scaling laws which were established in Refs. 8 and 9. The chaos arises initially as irregular oscillations in the signal level in the middle of the gently sloping regions. On an oscilloscope trace, this event corresponds to a sequential smearing of the elements of the structure which arose previously, in the order opposite that in which they appeared in the sub-critical region. With increasing λ the region occupied by the chaos and also the depth of the irregular oscillations increase.

Figure 2 is a map of the dynamic regimes in the plane of the control parameter X and the normalized delay time T/τ. Bifurcation lines found experimentally are shown here, along with those found numerically in a solution of Eqs. (2) and those found from the relations of Rets. 8 and 9 through the use of the universal function found there. That universal function determines corrections to the Feigenbaum bifurcation points for systems with a delay. At very large values of T/τ, the bifurcation values of λ go onto horizontal asymptotes, which correspond to bifurcation points of the mapping $f(U)$. With decreasing T/τ, the bifurcation lines bend upward; up to values T/τ~10, we find a good agreement with the data based on the universality and scaling relations,

When we return from the region of developed chaos back along the parameter λ, we frequently observe the appearance of regular regimes (oscillation modes), distinguished by a large number of the variations of the level over the delay interval (e and f in Fig. 1). At moderate values of T/τ (on the order of 15-20), the higher-order modes have a fundamental period which is smaller than the time required to traverse feedback loop by a factor of 3, 5,.... As λ is increased, these modes demonstrate a transition to chaos similar to that described above. The diagram remains the same in the λ, T/τ plane, but T must now be understood as half the period of the given oscillation mode. If T/τ is large, one can observe a tremendous variety of regimes on the return from the region of chaos to the region of regular dynamics. These regimes are characterized by differing numbers of samples and gently sloping regions of different lengths over the delay interval (Fig. 1f).

The system which we have studied here is not afflicted by the factors which complicate the analysis and interpretation of processes in conventional systems with delay (reflection of signals within circuit elements, etc.). On the other hand, it differs from the theoretical (numerical) models in that it allows one to bring the imposing arsenal of experimental research facilities to bear on the problem. Accordingly, despite the obvious disadvantage of a high level of digitization noise, we believe that this system is of definite interest.

1. V.Ya. Kislov, N.N. Zalogin, and E.A.Myasin, Radiotekh. Elektron. 24, 1118 (1979).
2. V. A. Kats and D. 1. Trubetskov, Pis'ma Zh. Eksp. Teor. Fiz. 39, 116 (1984) [JETP Lett. 39, 137 (1984)]
3. V. S. Anishchenko and D.E.Postnov. Zh. Tekh. Fiz. 55, 162 (1985) [Sov. Phys. Tech. Phys., 30, 91 (1985)]

4. A.S.Dmitriev and A.I.Panas, Zh. Tekh. Fiz. 56, 759 (1986)
5. R. Vallee and C.Delisle, Phys.Rev. A34, 309 (1986).
6. K. Ikeda, K.Kondo, O.Akimoto, Phys.Rev.Lett., 49, 1467 (1982)
7. M. Le Berre, E. Ressayre, A.Tallet, H. Gibbs, Phys.Rev.Lett., 56, 274 (1986)
8. S.P.Kuznetsov and A.S.Pikovsky. Izv. VUZov – Radiofizika, 26, 308 (1985)