Note that in the linear mode, amplitude and phase noise equal one another in intensity and each constitutes half the total noise. As is seen from Fig. 1, in the nonlinear mode amplitude noise may be neglected.

Figure 2 shows the noise factors in the presence of velocity, current, and field-noise (antenna-noise) fluctuations (K_{Nv}, K_{NI}, and K_{NF}), calculated according to the definition (11).

It is seen by comparing the curves of Fig. 2 that the largest contribution to the noise factor for given values of σ (by virtue of $C \ll 1$, and $|A_v|$, $|A_I| \approx 1$) are those of antenna noise and electron-velocity fluctuation noise. It is also seen that the total noise factor decreases with increasing values of the nonlinearity parameter D, to increase again near saturation (D = 3-4). Such a behavior is apparently due to the greater extent of small-noise amplification in the signal-saturation mode. Note also that the noise factor is a minimum for $b + y_1 \approx 0$.

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Stochastic Self-Oscillations and Instability in a Backward-Wave Tube

B.P. BEZRUCHKO, L.V, BULGAKOVA, S.P. KUZNETSOV AND D.I. TRUBETSKOV

Results of a computer solution of the nonstationary nonlinear equations of a backward-wave tube are shown, as well as data of experimental investigations, evidencing the fact that the transition from regular to stochastic self-oscillations corresponds to the arising of oscillation instability for small perturbations of the initial conditions.

* * *

INTRODUCTION

The system investigated — the backward type traveling-wave tube (BWT) — is a segment of an electrodynamic transmission line (a retarding system) matched at both ends and penetrated by an electron beam. The beam interacts with the wave, whose phase velocity v_p is close to the electron velocity v_0, with the group velocity v_g directed in an opposite direction to the

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beam. The output signal is derived from the same end of the line into which the beam is fired. If a number of simplifying assumptions is made, the dynamics of the process in such a system may be described by the equations [1]

$$\frac{\partial F}{\partial \tau}-\frac{\partial F}{\partial \xi}=-\frac{\mathscr{L}}{\pi}\int_0^{2\pi} e^{-i\theta_\alpha}\,d\alpha,\quad \frac{\partial^2\theta}{\partial\xi^2}=-\mathscr{L}^2\,\mathrm{Re}\,Fe^{i\theta_\alpha} \tag{1}$$

with the boundary and initial conditions

$$\theta_\alpha|_{\xi=0}=\alpha,\quad \left.\frac{\partial\theta_\alpha}{\partial\xi}\right|_{\xi=0}=0,\quad F|_{\xi=1}=0,\quad F|_{\tau=0}=F(\xi,0),$$

where F is the normalized field amplitude of the wave, θ_α is the electron phase coordinate relative to the wave, $\xi = x/l$ and $\tau=(t-x/v_0)/(l/v_0+l/|v_g|)$ are the normalized coordinate and time, and l is the system length. Equations (1) contain a single free parameter $\mathscr{L}=2\pi CN = C\Omega l/v_0$, where C is Pierce's parameter and Ω is the center spectral frequency of the signal generated. Note that $C\sim I^{1/3}U^{-1/2}$, where I is the electron-beam current and U the accelerating voltage.

As was shown earlier by numerical solution of Eqs. (1) [1, 2] and by experiment [2-4], as the parameter $\mathscr{L}$ increases, the BWT displays a sequence of increasingly complicated self-oscillatory modes, culminating in the onset of stochastic self-oscillations. As is well known, a similar behavior is also the case of many dynamical systems of various physical nature (see, e.g., [5-7]). According to present views, randomness arises when, for the given values of the parameters, all possible motions of the dynamical system are unstable for small perturbations [5-7]. It is such an interrelation between randomness and instability to constitute the object of the present investigation, by the example of the BWT.

1. EXPERIMENTAL RESULTS

The experiment consisted in observing the BWT output signal amplitude as a function of time while the electron beam was switched on and off in pulsed fashion. Owing to microfluctuations, an output signal realization with somewhat different initial conditions is observed on each switching of the beam. The diverging of these realizations with time may be used to assess the system stability in relation to small perturbations.

The experimental system — a decimeter-wave BWT laboratory model consisting of a retarding system of counterposing pins and grids traversed by a multibeam electron stream — was operated with a periodic sequence of rectangular beam-current pulses. The BWT output signal was applied to a high-speed oscillscope synchronized with the beam pulses. To ensure as far as possible the same initial conditions for each realization of the transient process, the retarding system was fed by an external multisinusoidal signal, several orders larger than microfluctuations but several orders smaller than the self-oscillations engendered. Oscillograms obtained for various values of the operating beam current are shown in Fig. 1, each of them resulting from the superimposition of 10^3-10^4 realizations of the process. In the oscillograms is discernible the BWT signal envelope, while the carrier oscillations are responsible for the luminous background. In Fig. 2 are shown the output-signal spectra for CW operating conditions of the tube for the same set of beam currents.

As may be seen from Figs. 1 and 2, regular self-oscillatory modes (those with a discrete spectrum) produce oscillograms that are sharply defined over their whole extent. This means that the microfluctuations present in the system do not cause any appreciable (discernible to the eye) variations in the realizations of the oscillatory process within the observation time. When however stochastic self-oscillations set in (then the spectrum is continuous), it is only the initial segment of the oscillogram to be clearly defined, which is evidence of the build-up of initially microscopic perturbations. The length of the initial clearly defined segment of the oscillogram, t_0, during which the difference between successive realizations becomes eventually macroscopic, is the limiting factor preventing in principle a deterministic description of the system dynamics for an existing level of microfluctuations.

If the initial amplitude-fluctuation level is due to the shot effect, its magnitude is given by $|F_i|\sim\sqrt{e\Omega C/I}\sim10^{-4}$–$10^{-5}$ [1, 3], where e is the electron charge. Assuming the perturbation build-up to be an exponential law, $|F|\sim|F_i|e^{h_p t}$, and considering that variations discernible to the eye must be in the order of $|F|\sim1$, we may estimate the order of magnitude of the Kolmogorov entropy h_p [5, 7], in dimensionless form

$$h_p\sim-\frac{1}{t_0}\ln|F_i|\sim\frac{10}{t_0}.$$

Table 1

I/I_t	$\mathscr{L}$	t_0	h_p, nsec^{-1}	h
20	6	170	0,06	1,8
30	7	150	0,07	2,1
100	10	80	0,12	3,6

Such estimates are summarized in Table 1 for various values of the ratio of operating current to the oscillation-onset current I_t. Alongside with h_p the table also lists the values of the dimensionless Kolmogorov entropy $h=h_p(l/v_0+l/v_g)$, embodying the same normalization as adopted in Eqs. (1).

2. NUMERICAL COMPUTER CALCULATIONS

The stability of the solution of Eqs. (1) was investigated numerically by the method described in [7]. Solutions were jointly obtained for two nonstationary boundary problems of type (1) with slightly differing initial conditions, both specified for the same instant of time τ_p (and accordingly qualified by the same p subscript)

$$F|_{\tau=\tau_p}=F_p^0(\xi),\quad F|_{\tau=\tau_p}=F_p(\xi)=F_p^0+\tilde{F}_p(\xi).$$

The complex functions $F_p^{(0)}(\xi)$ and $F_p(\xi)$ describe the field distribution in the BWT in the absence and in the presence of initial perturbation, the norm of the latter being small, $\|\tilde{F}_p\| = \left[\int_0^1 |\tilde{F}_p(\xi)|^2 d\xi\right]^{1/2}=\varepsilon\ll 1$. As a result of solution of Eqs. (1), we obtain two new functions $F_{p+1}^0(\xi)$ and $F'_{p+1}(\xi) = F_{p+1}^0(\xi)=\tilde{F}'_{p+1}(\xi)$, both corresponding to the instant of time $\tau_{p+1}=\tau_p+\tau_1$. The interval τ_1 is assumed to be small, and the perturbation to have remained small: $\varepsilon_{p+1}= \|\tilde{F}'_{p+1}\|\ll 1$. We now renormalize the perturbation $\tilde{F}_{p+1}$ so that its norm shall again equal ε:

$$\tilde{F}_{p+1}(\xi)=\frac{\varepsilon}{\varepsilon_{p+1}}\tilde{F}'_{p+1}(\xi),$$

$$F_{p+1}(\xi)=F_{p+1}^0(\xi)+\tilde{F}_{p+1}(\xi).$$

The calculation is then repeated with the new initial conditions $F|_{\tau=\tau_{p+1}}=F_{p+1}^0(\xi)$ and $F|_{\tau=\tau_{p+1}} = F_{p+1}(\xi)$, and so on*. Clearly, the solution $F^0(\xi,\tau)$ is stable with respect to the perturbation considered if the value of $D(p)=\prod_{m=0}^{p}(\varepsilon_m/\varepsilon)$ is limited, and it is unstable if $D(p)$ increases with increasing values of p. The Kolmogorov entropy is estimated from the expression

Table 2

$\mathscr{L}$	5,5	6,25	7
h	0,5	1,6	1,8

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*To avoid calculation errors building up in the iterative procedure, the perturbation was systematically purged of any contributions corresponding to a time (and phase) shift of the solution. To this end, the $(p+1)$-th perturbation was recalculated as $\tilde{F}''_{p+1}(\xi)=\tilde{F}'_{p+1}(\xi)-ic_1F_{p+1}^0(\xi)-c_2(\partial F_{p+1}^0/\partial\tau)$, where c_1 and c_2 are real numbers selected to minimize $\|\tilde{F}''_{p+1}\|$.

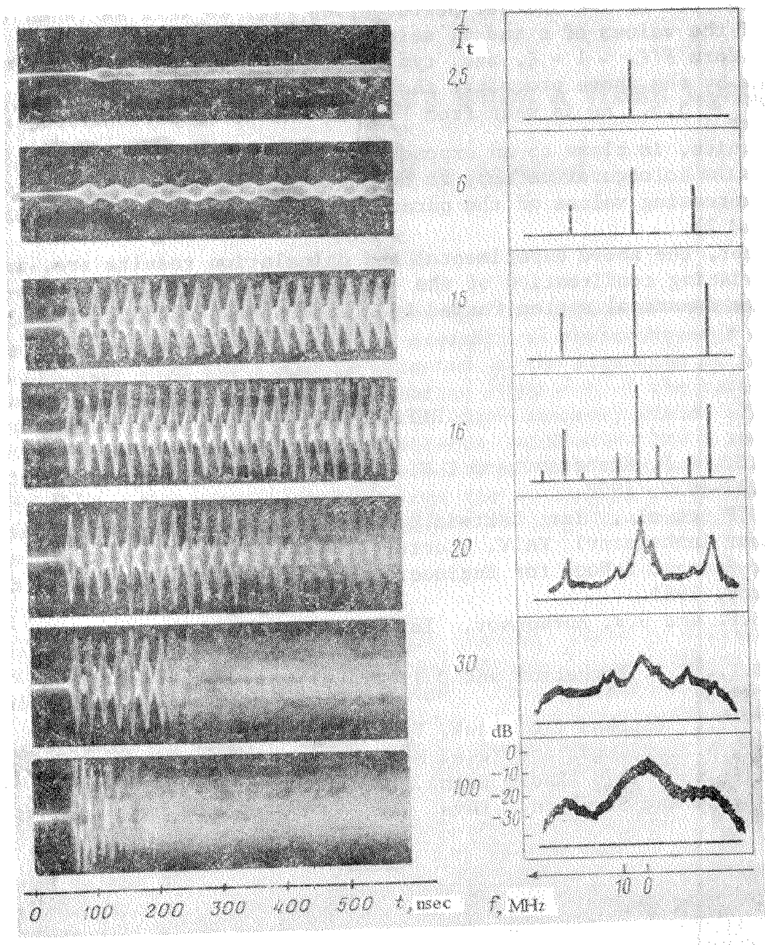

Fig. 1. Oscillograms of BWT output-wave envelope showing oscillation transients for various I/I_t ratios.

Fig. 2. BWT output-signal steady-state spectra for various I/I_t ratios.

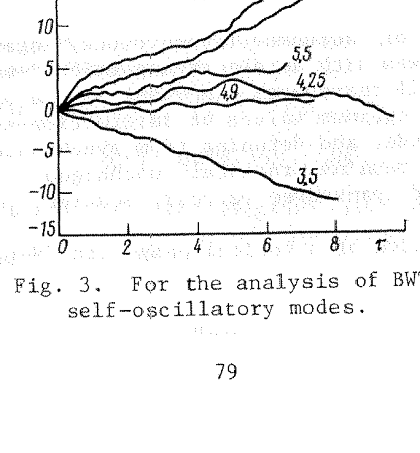

Fig. 3. For the analysis of BWT self-oscillatory modes.

$k \sim \ln D(p)/p\tau_1$. Note that the character of the behavior of $D(p)$ is found to be practically independent of the form of the initial perturbation $\tilde{F}(\xi)$ as well as (when the above conditions are satisfied) of the values of ε and τ_1 selected. As a rule, the inital perturbation was specified in the form $F(\xi) \sim 1 - \xi$, and, typically, $\varepsilon = 0.01$ and $\tau_1 = 0.2$.

Calculations by the above procedure show that instability of a self-oscillatory mode arises for $\mathscr{L} > \mathscr{L}_{cr} \approx 5.5$. As is seen from Fig. 3, the perturbation rise, averaged over a time τ of a few units, is close to an exponential law. The limited length of the realizations available enables the Kolmogorov entropy to be estimated by only its order of magnitude. It increases with increasing values of the parameter $\mathscr{L}$ and is close to the experimental estimate (cfr. Tables 1 and 2).

Taken together, the above experimental and calculation results are, in the authors' view, fairly convincing confirmation of the interrelation between stochastic behavior and instability in the dynamical system formed by electron beam and backward electromagnetic wave in a BWT.

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Stimulated Cherenkov Radiation by Ultrarelativistic Helical Electron Beams

N.F. KOVALEV

It is shown that for Cherenkov (synchronous) interaction of an elementary (thin) electron beam with an electromagnetic TM-wave, the occurrence of a comparatively small transverse component in the electron velocity substantially affects the optimum values of interaction-space length, high-frequency field amplitude, and detuning from synchronism; at the same time the maximum efficiency remains practicaly unchanged. However, the arising in the electron beam of transverse velocity scatter, lowers the efficiency of microwave devices.

Cherenkov interaction of a helical beam with TW-waves is also considered.

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