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Reconstructions of model equations of time-delay system from short experimental time series

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This study aims to investigate the scope of methods for the reconstruction of time-delay systems. We consider an approach to the reconstruction of time-delay systems based on the synchronous response of the driven system with the structure similar to the structure of the studied object. This approach is used for the recovery of the parameters of time-delay systems from short and noisy time series. To show the operational performance and capabilities of this approach, the parameters were reconstructed for a radiophysical chaotic generator with quadratic nonlinearity and for the model of a system for the baroreflectory regulation of the mean arterial pressure.

Keywords: Time-delay systems; reconstruction; chaos; radiophysical experiment.

1. Introduction

Dynamic reconstruction of mathematical models of systems from experimental data reveals vast opportunity, allows one to develop the fundamental understanding of

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the internal structure of the studied systems, and to evaluate the parameter values unavailable for direct measurement.[1] Traditional approaches to model reconstruction from experimental data are often ineffective and do not reveal the model structure. In these cases, some sensible assumptions on the structure of the system equations and parameter evaluations can be made based on these time series. Such cases require the development of specialized methods oriented to narrow classes of systems.[2,3] One of these interesting equation classes is time-delay equations. Self-oscillating models of delay systems — time-delay generators — are characteristic of many technical, live systems and other real-world objects.[4–7]

The reconstruction of systems functioning in near-periodic modes presents an additional difficulty, since such signals convey less information about the system than the chaotic time series, requiring longer signals for analysis.

This paper suggests a specialized approach of reconstructing time-delay generator models. This approach is based on the construction of a driven system with the structure similar to the one of the studied system, and on the observation of its synchronous response to the experimental signal. The capabilities of this approach in the analysis of short time series are studied in the course of the reconstruction of experimental setup mathematical model — a hybrid chaotic time-delay generator, as well as periodic time series of mathematical models of the system of biological nature.

2. Material and Methods

2.1. *Methods*

The paper considers the approach targeted at reconstructing the parameters of the systems described by the following equation:

$$\varepsilon_0 \dot{x}(t) = -x(t) + f(x(t-\tau_0)), \tag{1}$$

where τ_0 is a delay time, ε_0 is a parameter of inertia, and f is a nonlinear function.

We suggest a method of reconstructing the delay time based on the observation of the synchronous response of the driven system. The studied systems time series $x(t)$ is fed to the input of the driven system structurally identical to the mathematical model of the studied system but with the feedback loop broken by the subtractor introduced into the feedback chain. The equation of receiver dynamics is as follows:

$$\varepsilon_0 \dot{y}(t) = -y(t) + f(x(t-\tau_0)). \tag{2}$$

The subtractor output yields the recovered useful signal as $z(t) = f(x(t-\tau_0)) - f(y(t-\tau_0))$. If the elements of the receiver and transmitter are identical, the two systems will be synchronized after a certain transient process. Indeed, the difference $\Delta(t) = x(t) - y(t)$ between oscillations of systems in Eqs. (1) and (2) decreases with the time for any $\varepsilon > 0$, since $\dot{\Delta}(t) = -\frac{\Delta(t)}{\varepsilon_0}$. As a result of this synchronization, we have $x(t) = y(t)$ and, hence, $x(t-\tau_0) = y(t-\tau_0)$ and $z(t) = 0$ (Fig. 1).

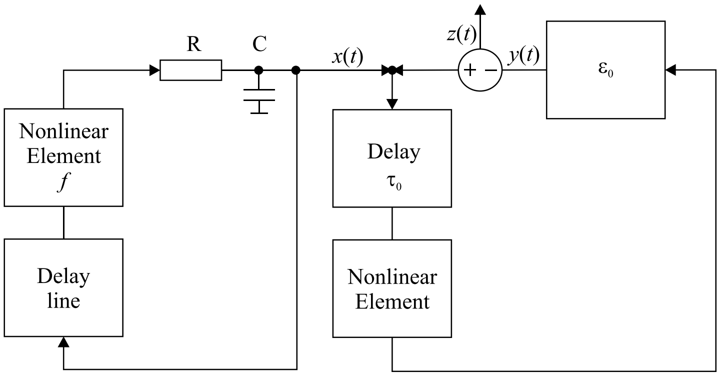

Fig. 1. The operating principle of the applied reconstruction approach is based on the auxiliary system's synchronous response.

If the parameters of the driven system are identical to those of the studied system, the dispersion D of subtractor/difference signal $z(t)$ will be defined purely by the measurement noises and will equal 0 in the case of their absence. If the parameters of the driven system are different, the dispersion of $z(t)$ of the difference signal will reach the dispersion values of the experimental signal itself. A similar approach has been used in reconstructing the chaotic system of hidden transmission of information suggested in Ref. 8.

To solve the reconstruction task, the parameters of the nonlinear function $\mathbf{a}$ and control parameters τ and ε are determined by minimizing the target function — dispersion $D(\tau, \varepsilon, \mathbf{a})$ of the driven system output subtractor signal.

2.2. *Studied systems*

The paper studies the time series of a radiophysics setup — chaotic time-delay generator — implemented as a hybrid device. Delay line and nonlinear block were implemented digitally on the basis of 32-bit ARM microcontroller (MC) Atmel ATSAM3X8E, the nonlinear element output signal was fed to the input of the 16-bit DA converter (DAC) Analog Devices AD5060, it passed through the time-delay/inertial block featuring the RC low-pass filter (LPF), and was digitalized by 16-bit AD converter (ACD) Analog Devices ADS8326. Digitalized signal was then fed to the input of the delay line featuring a circular buffer in the MC random access memory (Fig. 2). At the input and output of LPF, repeating amplifiers were placed performed on the precision operational amplifier Analog Devices AD822. The dynamic variable observed at LPF output was digitalized by the 16-bit AD converter National Instruments PXIe-6355 with the rate of 0.1 MS/s and then was stored on a PC for further reconstruction.

The operation of ADC, and DAC and digital measurements of the MC were synchronized by the interruption of its precision 32-bit timer. The MC core was clocked 84 MHz with the clock signal (max rate for this MC) from 20 MHz of

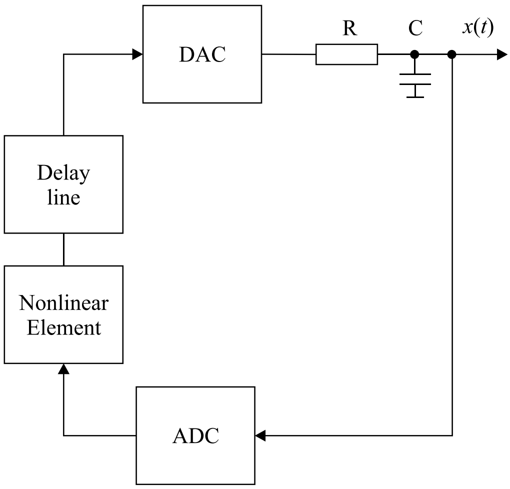

Fig. 2. Block diagram of the experimental setup for studying chaotic time-delay generator with quadratic nonlinearity.

the quartz resonator via the phase-locked loop. The setup performed one cycle of numerical calculations with the data update at the DAC output and ADC input in $\Delta t = 10^{-5}\,\mathrm{s}$.

Quadratic transformation was chosen as transfer-function for nonlinear block of time-delay generator. Thus, this time-delay generator can be described as

$$RC\dot{x}(t) = -x(t) + \lambda_0 - x^2(t - \tau_0), \tag{3}$$

where λ is a parameter of nonlinearity and $RC = \varepsilon_0$ is a response time of LPF. Equation (2) can be transformed into Eq. (1) by the following changes: $\varepsilon_0 = RC$, $f(t - \tau_0) = \lambda - x^2(t - \tau_0)$. During our experiments time-delay generator had the following parameters: $R = 5358.8\,\mathrm{Ohm}$, $C = 46.6\,\mathrm{nF}$, $\varepsilon_0 = 203\,\mathrm{ms}$, $\tau_0 = 2500\,\mathrm{ms}$ (250 delay times), and $\lambda_0 = 1.74\,\mathrm{V}$.

To exclude the influence of quantization noises, measurement noises and distortions from analog components, the results of recovering the radiophysical generator model from experimental data were compared to the results obtained from recovering mathematical model of the identical time-delay generator.

To study the possibilities of the reconstruction method in the analysis of short periodic time series, we have chosen the model of baroreflex regulation of mean arterial pressure suggested in Ref. 6 The model equation of this system is based on the results of physiological experiments and is written as in Eq. (1) with sigmoidal nonlinear function f

$$f(x) = k\left(\frac{r^*}{1 + \alpha\varepsilon^{-}\beta x} - \frac{r^*}{1 + \alpha\varepsilon^{\beta} x}\right), \tag{4}$$

where $\alpha = 1$, $\beta = 2$, $r^* = 1$, and $k = -1.65$ are parameters whose values were chosen in Ref. 4 to better approximate the results of the physiological experiment.

To decrease the number of fitting free parameters during reconstruction, function from Eq. (4) was approximated as

$$f(x) \approx a_0 \tanh(b_0 x), \tag{5}$$

where $a_0 = -1.65$ and $b_0 = 1$ are free parameters. The chosen parameters in Eq. (4) provide the best least-squares approximation of function in Eq. (4).

The nonlinear function is a sigmoidal one and the system can only perform periodic oscillations. With parameter values $\tau_0 = 3.6\,\mathrm{s}$ and $\varepsilon_0 = 2.0\,\mathrm{s}$ typical for healthy people, the system exhibits periodic oscillations with the period of about 10 s, which correlates well with the experimental observations. The method of Euler with the integration step of 0.01 was used to obtain time series from Eq. (1) with nonlinear function from Eq. (4).

The studies of real systems are always complicated by the presence of measurement noises. Therefore, during the numerical simulation we investigated the possibility of signal reconstruction in the presence of Gaussian measurement δ-correlated noise.

2.3. *Results*

Figure 3 presents parts of time series and power spectra of experimental radiophysical setup and mathematical model described by Eq. (3). Standard deviations for experimental and model signals were 0.8 V and 0.81 V, respectively. Therefore, approximately 1% of standard deviation of experimental signal is defined by noises of various origins.

Assuming that the equation structure in the form of Eq. (3) is *a priori* known, we carried out the dynamic reconstruction of the parameters of model equations from obtained chaotic time series. Cross-sections of $D(\tau, \varepsilon, \lambda)$ function for each parameter at the point which corresponds to the minimal value of D for the reconstruction of the time-delay generator, are presented in Fig. 4 for both radiophysical and numerical experiments.

With the help of the reconstruction procedure, we recovered the following parameter values from the time series of numerical model: $\tau^M = 2500\,\mathrm{ms}$, $\varepsilon^M = 203\,\mathrm{ms}$, $\lambda^M = 1.74\,\mathrm{V}$ (the relative errors of the parameters estimation are 0.01%, 0.02%, 0.03%, respectively), and from experimental data: $\tau^E = 2490\,\mathrm{ms}$, $\varepsilon^E = 208\,\mathrm{ms}$, $\lambda^E = 1.78\,\mathrm{V}$ (the relative errors of the parameters estimation are 0.01%, 0.02%, 0.03%, respectively). We found out that 0.004 s long time series (400 points of discrete time or 1.5 delay times) are sufficient to recover the parameters of both experimental and numerical systems. A further increase of time series' length did not enhance the parameter recovery accuracy.

The opportunities of the methods applied in the reconstruction of time-delay system models from short time series were studied for the case of reconstruction of the baroreflex regulation model of mean arterial pressure with and without measurement noises. Parts of time series and power spectra of this model without noise

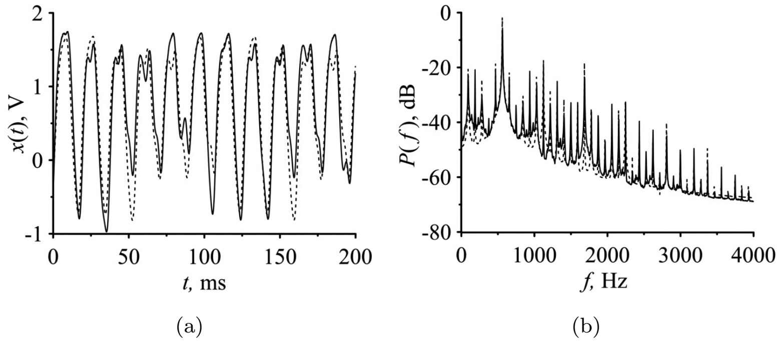

Fig. 3. An example of signals and their spectra of the experimental radiophysical setup and its mathematical model: (a) time series (mean-square error is 0.09) and (b) power spectra of experimental setup (solid line) and model of time-delay generator (dashed line).

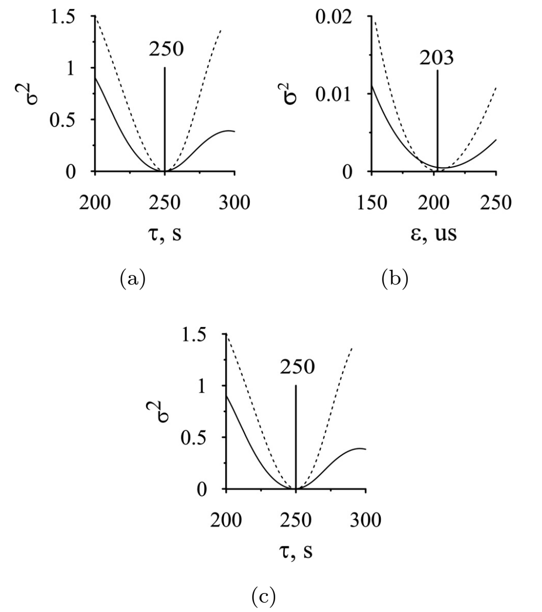

Fig. 4. Cross-sections of $D(\tau, \varepsilon, \lambda)$ function at the point corresponding to the minimal value of D for parameters: (a) τ, (b) ε, (c) λ. The cross-sections were calculated from time series of radiophysical chaotic generator (solid line) and noiseless model of time-delay generator (dashed line).

and with 1% noise (in terms of noise dispersions and noiseless signals ratio) are presented in Fig. 5.

The results of reconstruction in the form of cross-sections of $D(\tau, \varepsilon, a, b)$ function for each parameter at the minimum point D are represented in Fig. 6. From noiseless time series, we recovered the following parameter values: $\tau^c = 3.59\,\text{s}$, $\varepsilon^c = 2.00\,\text{s}$, $a^c = -1.65$, and $b^c = 1.00$. Those values can be recovered with the time series length of more than 20 s (2000 points of discrete time — or 5.5 delay times). For the model equation in 1% measurement noise we obtained the following parameter values: $\tau^n = 3.59\,\mathrm{s}$, $\varepsilon^n = 2.00\,\mathrm{s}$, $a^n = -1.65$, and $b^n = 1.00$. The reconstruction can be accomplished with time series of over 30 s (3000 points of discrete time or 8.5

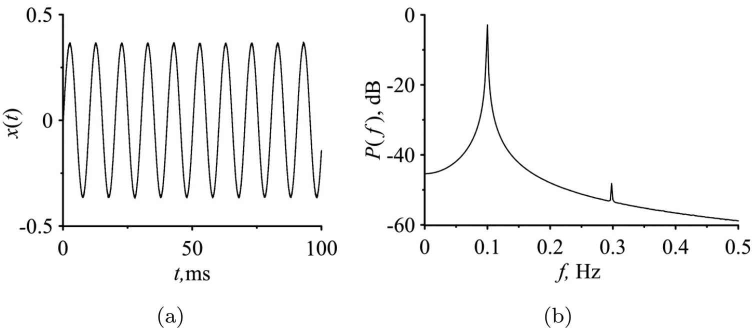

Fig. 5. An example of signals and their spectra of the baroreflex regulation model: (a) time series (mean-square error is 0.00001) and (b) power spectra of the model of baroreflectory regulation of arterial pressure without noise (solid line) and in the presence of 1% Gaussian noise (dot line).

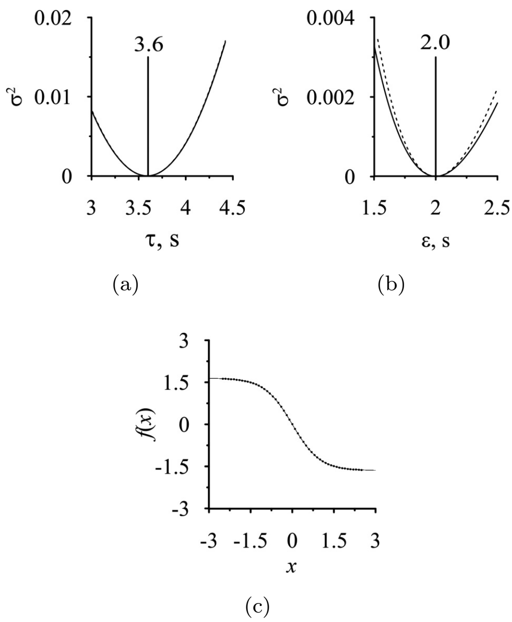

Fig. 6. Cross-sections of $D(\tau, \varepsilon, \lambda)$ functions for parameters τ (a), ε (b), while other parameters correspond to the minimal value of D. The cross-sections were calculated from time series of noiseless model of baroreflectory regulation of mean arterial pressure (dashed line) and from model with 1% Gaussian noise (solid line). The cross-sections were made with the step of the parameters τ and ε variation equal to 0.01. (c) The nonlinear function equation (3) of the system of biological nature (dashed line) is compared to the recovered nonlinear function equation (4). The reconstruction was carried out from a short time series with the length of 3000 discrete samples (3 characteristic oscillation periods), in the presence of 1% measurement noise.

delay times). In the case of smaller length of time series, the parameter estimation error dramatically increases. A further increase of time series length does not result in the enhancement of parameter recovery accuracy.

The reconstruction of the nonlinear function equation (5) from noisy time series is compared to the reference function equation (4) in Fig. 6(c). Clearly, the nonlinear function is reconstructed with high accuracy despite the presence of noise.

3. Discussion

Due to this fact, a range of specialized reconstruction methods has been suggested in recent years.[9–17] However, in a number of cases, these methods fail as well, and a search of new approaches is in high demand. One of the cases requiring the development of specialized approaches is the limited observation time of the time series of dynamic variable. This is typical for the signal analysis of many technical systems, in particular, communication systems,[8] and it is particularly characteristic of live systems.[6] Live systems are, as rule, highly nonstationary objects, whose parameters may quickly and significantly change in time and with the change of external conditions. In all these instances essentially the only solution is to conduct reconstructions from short time series.

Traditional methods of chaotic system reconstruction, which do not take into account the *a priori* information of the studied system, generally require the time series of 10 and longer delay time,[9–15] which limits the opportunities of using them in analyzing experimental data.

This paper focuses on solving the problem of reconstructing periodic and chaotic time-delay systems from short time series. Specialized methods of reconstruction are used for this purpose. They are based on the synchronous response of the driven system to the experimental time series, the structure of this system being similar to the structure of the model equation of the studied object.

The obtained results indicate that the proposed approach takes into account the *a priori* information about the studied object and allows one to recover the model parameters with good accuracy from shorter time series than other known approaches. Since the chaotic signals contain more information than periodic signals, the chaotic generator parameters can be recovered from even shorter time series (less than 2 delay times) than from periodic (about 10 delay times in the presence of 1% measurement noise).

4. Conclusion

Model equations of radiophysical chaotic delay generator have been recovered, as well as its numerical model, and the model of the system of biological nature with periodic dynamics. It has been shown that the measurement noise in time series increases the requirements for the time series minimum length necessary for the successful parameter reconstruction.

Acknowledgments

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